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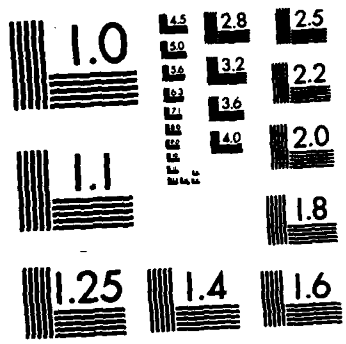

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THEORIES OF KINEMATIC ANALYSIS AND SYNTHESIS OF SPATIAL MECHANISMS CONTAINING LOWER AND HIGHER PAIRS

FINAL REPORT
Period Covered 7/20/81 - 7/19/83

Dr. George N. Sandor, P.I.,
Dr. Dilip Kohli, Dr. Charles F. Reinholtz,
Mr. Ashitova Ghosal, Dr. Manuel Hernandez
and Mr. Partha De, with the help of Mr. Martin DiGirolamo

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Kinematics, Synthesis, Analysis, Higher Pairs, Mechanism, Spatial Mechanisms, Mechanism type, Sphere-plane pairs, Sphere-grove pairs, Sphere-slotted-cylinder pairs, Branch avoidance order defect, Grashof analysis, Mobility analysis, Position stacking, Wholistic design, Non-linear equations, Optimization, Path generation, Function generation, Motion generation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Accomplishments to date include vector-theories for the analysis of spatial function, path, and motion generators containing higher-pair joints. Also completed are design theories which assure that a synthesized mechanism is free from branching defects. Additional theories have been developed for synthesizing several types of single-input spatial motion generator mechanism with complete input crank rotation, optimal transmission characteristics, and correct order of output positions. Methods have also been developed for efficently formulating and solving systems of non-linear

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equations which commonly arise in the synthesis of spatial mechanisms. The theories developed under the sponsorship of this grant have expanded the utility of spatial mechanisms. It has led to simplified analysis and design theories for spatial mechanisms containing higher pairs and it has produced a new "wholeistic" approach to spatial.

4. List of Material Contained in the Appendix

The appendix contains copies of the following papers which have been published. The titles and authors have been listed below, all other publication material is contained in section 5c. of this report.

1) "Kinematic Analysis of Three-Link Spatial Mechanisms Containing Sphere-Plane and Sphere-Groove Pairs," G.N. Sandor, D. Kohli, M.V. Hernandez, and A. Ghosal.

2) Kinematic Analysis of Four-Link Space Mechanisms Containing Sphere-Groove and Sphere-Slotted-Cylinder Higher Pair," A. Ghosal, D. Kohli, and G. N. Sandor.

The abstracts of the following Masters Thesis and Doctoral Dissertations have also been included in the appendix.

1. "Analysis of Spatial Mechanisms Containing Higher Pairs," Masters Thesis by Ashitava Ghosal.

2. "Optimization of Spatial Mechanism", Ph.D. Dissertation by Charles F. Reinholtz.

3. "Kinematic Synthesis and Analysis of Three-Link Spatial Function Generators with Higher Pairs," Ph.D. Dissertation by Manual V. Hernandez.

TABLE OF CONTENTS

5.a. State of the Problem Studied

Design and analysis theories for planar mechanisms are well developed and such devices are in common use. However, many automation tasks require mechanisms which can generate spatial motion. One solution is to employ multi-degree-of-freedom, multiple-input robotic manipulators. However, these devices are limited in speed and accuracy, and require sophisticated electronic control systems. On the other hand, sigle-input spatial mechanisms, the topic of this research, are purely mechanical, and are better suited for performing highly repetitive automation tasks of limited complexity more efficiently, reliably and economically than robotic manipulators.

Single-input spatial mechanisms are much more difficult to design and analyze than planar mechanisms. As a result, their use to date has been quite limited. This is especially true of spatial mechanisms containing higher pairs (joints which develop only point or line contact and allow several degrees of freedom of relative motion).

The research being conducted under this grant attempted to develop simplified theories for designing and analyzing single-input spatial mechanisms.

5.b. Summary of Most Important Results

Accomplishments to date include vector-theories for the analysis of spatial function, path and motion generators, containing higher-pair joints which allow minimizing the number of mechanical parts. For example, a newly analyzed class of spatial function generators has only two moving links: the input and the output. Also completed are design theories which assure that a synthesized mechanism is free from the "branching defect" (i.e. satisfies the physical motion requirements as well as the mathematical criteria. Additional theories have been developed for synthesizing several types of single-input spatial motion generator mechanisms to have complete input crank rotation, to have optimal transmission characteristics and to have the correct order of output positions.

Methods have been developed for efficently formulating and solving systems of non-linear equations which commonly arise in the synthesis of spatial mechanisms.

It is believed that the theories developed under the sponsorship of this grant have greatly expanded the utility of spatial mechanisms in two important ways. First, it has led to simplified design and analysis theories for spatial mechanisms containing higher pairs. Second, it has produced a new "wholeistic" approach to spatial mechanism design, wherein many of the "real-world" constraint conditions are considered in the design process.

5.c. List of Publications

1. "Kinematic Analysis of Three-Link Spatial Mechanisms Containing Sphere-Plane and Sphere-Groove Pairs" G.N. Sandor, D. Kohli, M.V. Hernandez and A. Ghosal, Proceedings of the Seventh Applied Mechanisms Conference, 1981 pp. XXXII-1 to XXXII-11; Mechanism and Machine Theory, 1984.

2. "Kinematic Analysis of Four-link Space Mechanisms Containing Sphere-Groove and Sphere-Slotted-Cylinder Higher Pairs," A. Ghosal, D. Kohli, and G.N. Sandor, ASME paper 82-DET-123, Presented at the 1982 ASME Mechanisms Conference.

3. "Analysis of Spatial Mechanisms Containing Higher Pair," Masters Thesis by Ashitava Ghosal, Presented to the Graduate Council of the University of Florida, August, 1982.

4. "Optimization of Spatial Mechanisms," Ph.D. Dissertation by Charles F. Reinholtz, presented to the Graduate Council of the University of Florida, August, 1983.

5. "Kinematic Synthesis and Analysis of Three-Link Spatial Function Generators with Higher Pairs," Ph.D. Dissertation by Manuel V. Hernandez, Presented to the Graduate Council of the University of Florida, April 1983.

5.d. Participating Scientific Personnel

Personnel Drawing Support from this Project:

1) Dr. George N. Sandor, P.I.
2) Dr. Dilip Kohli, Consultant
3) Mr. Ashitava Ghosal, earned Ph.D., August, 1983.
4) Dr. Charles Reinholtz, earned Ph.D., August, 1983.
5) Mr. Partha De, Master's Degree Candidate

Personnel Contributing to the Research but not drawing support from this Project:

1) Mr. Xirong Zhuang, Visiting Engineer from the People's Republic of China.
2) Dr. Manuel V. Hernandez, earned Ph.D., May, 1983.

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7. APPENDICES

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KINEMATIC ANALYSIS OF THREE-LINK SPATIAL MECHANISMS CONTAINING SPHERE-PLANE AND SPHERE-GROOVE PAIRS

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Abstract—Kinematic pairs in a spatial mechanism are viewed either as allowing relative screw motion between links or as constraining the motion of the two chains of the mechanism connected to the two elements of the pair. Using pair geometry constraints of the sphere-plane and sphere-groove kinematic pairs, the displacement, velocity and acceleration equations ae derived for, *R–Sp–R*,¶ *R–Sp–P*, *P–Sp–P*, *P–Sp–R* and *R–Sg–C* three-link mechanisms. For known values of the input variable, other variables are computed in closed form. The analysis procedures are illustrated using numerical examples.

1. INTRODUCTION

The mechanisms containing higher pairs such as cams, sphere–plane, sphere–groove, or cylinder–plane provide the designer with the capabilities of designing machines and mechanisms to satisfy more complex and exact functional requirements than feasible with only lower pair mechanisms. These mechanisms in general are compact and contain fewer links than those with lower pairs.

In recent years, there has been considerable development in the tools for kinematic analysis of spatial mechanisms containing lower pairs.

Kinematic analysis of space mechanisms was initiated by the significant contribution of Dimentberg[1]. Dimentberg[2, 3] demonstrated the use of dual numbers and screw calculus to obtain closed-form displacement relationships of an *RCCC*¶ and other four-, five-, six- and seven-link spatial mechanisms containing revolute, cylinder, prismatic and helical pairs. Denavit[4] derived closed-form displacement relationships for a spatial *RCCC* mechanism using dual Euler angles. Yang[5] also derived such relationships for *RCCC* mechanisms using dual quaternions.

Vectors were first used by Chace[6] to derive closed-form displacement relations of *RCCC* mechanisms. Wallace and Freudenstein[7] also used vectors to obtain closed-form displacement relations of *RRSRR* and RRP_LRR mechanisms.

Yang[8] proposed a general formulation using dual numbers to conduct displacement analysis of *RCRCR* spatial five-link mechanisms. Soni and Pamidi[9] extended this application of (3 × 3) matrices with dual elements to obtain closed-form displacement relations of *RCCRR* mechanisms.

Yuan[10] employed screw coordinates to obtain closed-form displacement relations for *RRCCR* and othe spatial mechanisms.

Jenkins and Crossley[11], Sharma and Torfason[12], Dukkipati and Soni[13] used the method of generated surfaces to conduct analysis of single loop mechanisms containing revolute, prismatic, cylinder, helical and spheric pairs. Hertenberg and Denavit[14] contributed iterative techniques to conduct displacement analysis of spatial mechanisms using (4 × 4) matrices containing revolute, prismatic, cylinder, helical and spheric pairs. Uicker[15] explored in further detail the (4 × 4) matrix approach of Hartenberg and Denavit. Soni and Harrisberger[16] contributed an iterative approach for performing kinematic analysis using (3 × 3) with dual elements. Kohli and Soni[17, 18] used finite screws to conduct displacement analysis of single-loop and two-loop space mechanisms involving *R*, *P*, *C*, *H* and *S* pairs.

Bagci[19] used a (3 × 3) screw matrix for displacement analysis of a mechanism containing two revo-

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¶*R*: revolute, *P*: prismatic, *C*: cylindric, *S*: spherical, *Sp*: sphere-plane and *Sg*: sphere-groove joint.

lute pairs, one cylinder pair and one spheric pair. Dobrovolski[20] used the method of spherical images to analyze space mechanisms containing revolute and cylinder pairs. Duffy[21, 22], Duffy and Habib-Olahi[23] used the method of spherical triangles to derive displacement relations for five and six link mechanisms containing revolute and cylinder pairs. Keller[25] and Gupta[26] also analyzed space mechanisms containing revolute, prismatic, cylinder, helical and spheric pairs. Recently Kohli and Soni[26] and Singh and Kohli[27] used the method of pair constraint geometry and successive screw displacements to conduct analyses of single and multi-loop mechanisms.

In the present paper, screw displacements expressed in vector form and the pair geometry constraints, also expressed in vector form, are used to derive the displacement, velocity and acceleration equations for *R–Sp–R*, *R–Sp–P*, *P–Sp–R*, *P–Sp–P* and *R–Sg–C* three link mechanisms.

Since Revolute (*R*) and Prismatic (*P*) pairs are special cases of the cylinder pair (in prismatic pairs, the rotation is zero; for revolute pairs sliding is zero), we derive the analysis equation for *C–Sp–C* and *C–Sg–C* mechanisms, and then force rotations or translations at one or more pairs to zero, to obtain the equations for the above described three-link one degree of freedom mechanisms.

Briefly, the procedure for obtaining the analysis equations is as follows.

Step 1. Consider the *C–Sp–C* mechanism and the *C–Sg–C* mechanism.

Step 2. Separate the two moving links (Bodies 1 & 2) at the sphere-plane pair for the *C–Sp–C* case and at the sphere-grove pair for the *C–Sg–C*.

Step 3. Use the screw displacements in vector form to describe the new (*j*th) position of the sphere-plane (*Sp*) or sphere-groove (*Sg*) pairs from two sides of the pair.

Step 4. Use the pair geometry constraints on the position of the pair obtained from two sides.

Step 5. Force the cylindrical (*C*) joints as revolute (*R*) or prismatic (*P*) joints by setting the sliding or the rotation equal to zero at cylindrical pairs.

2. THE THREE-LINK MECHANISM AND ASSOCIATED VECTORS

Figure 1 shows the initial position of two rigid bodies grounded via cylindrical pairs and connected together by a sphere-plane pair. Also shown are the following vectors and scalar quantities:

- $\mathbf{u}_A$ unit vector defining the direction of the axis of cylindric pair *A*.
- $\mathbf{u}_B$ unit vector defining the direction of the axis of cylindrical pair *B*.
- **P** vector locating the axis of cylindric pair at *A* in the fixed coordinate system.
- **Q** vector locating the axis of cylindric pair at *B* in the fixed coordinate system.
- **A** unit vector perpendicular to the plane of the *Sp* pair embedded in body 1.
- **A′** vector embedded in body 2, congruent with **A** in the starting position, as shown in Fig. 1.
- **R** vector locating point *R*, the sphere center in the fixed coordinate system.
- θ_A rotation of link 1 about axis $\mathbf{u}_A$.
- θ_B rotation of link 2 about axis $\mathbf{u}_B$.
- S_A translation of link 1 along axis $\mathbf{u}_A$.
- S_B translation of link 2 along axis $\mathbf{u}_B$.

Figure 2 shows the *C–Sg–C* mechanism with all associated vectors and scalars. Description of all parameters are the same as for the *C–Sp–C* mechanism except for the direction of the vector **A**, which is now along the direction of the groove and also the addition of S_G, which is the translation of the sphere along the direction of **A**.

3. PAIR GEOMETRY CONSTRAINT EQUATIONS

Figures 3 and 4 show a sphere-plane (*Sp*) pair and a sphere-groove (*Sg*) pair with the vector **R** locating *R*, the sphere center. The vector **A**, in the *Sp* pair is defined as a vector perpendicular to the plane in which the sphere moves. In the *Sg* pair, the vector **A** defines the direction of the groove.

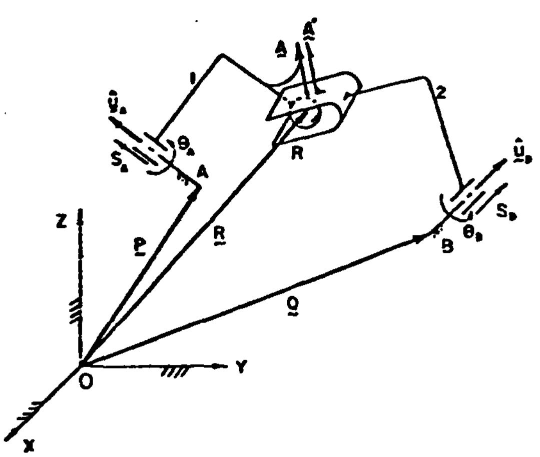

Fig. 1. C-S_p-C mechanism.

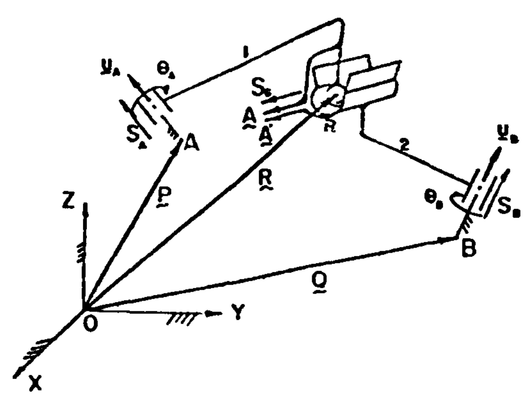

Fig. 2. C-S_g-C mechanism.

We can now define the vectors $\mathbf{R}_j$, $\mathbf{A}_j$, $\mathbf{R}_j'$ and $\mathbf{A}_j'$. These new vectors will define the displaced position and direction of initially coincident point R and vector $\mathbf{A}$ in bodies 1 and 2 respectively after some relative motion between bodies 1 and 2. The prime notation here is used for new position expressed from the motion of body 2, whereas the unprimed notations are used for new positions expressed from the motion of body 1.

The pair geometry constraint equation for the Sp pair is†

$$\frac{d^n}{dt^n}[(\mathbf{R}_j - \mathbf{R}_j') \cdot \mathbf{A}_j'] = 0, \quad n = 0, 1, 2, \ldots \tag{1}$$

which expresses that any relative motion between the sphere and the plane must be perpendicular to the vector $\mathbf{A}_j'$ (Fig. 1).

The pair geometry constraint equation for the Sg pair is

$$\frac{d^n\mathbf{R}_j}{dt^n} - \frac{d^n\mathbf{R}_j'}{dt^n} = \frac{d^n}{dt^n}(\mathbf{A}_j' S_{Gj}) \quad n = 0, 1, 2, \ldots \tag{2}$$

where S_{Gj} is the translation of the sphere along the groove in the direction of $\mathbf{A}_j'$. The constraint equation for the Sg pair expresses that any relative motion between the sphere and the groove must be along the groove which is in the direction of $\mathbf{A}_j'$ (Fig. 2).

4. WORKING EQUATIONS

Referring to Fig. 1, let $\mathbf{A}$ be a vector in body 1 $\mathbf{A}'$ a momentarily congruent vector in body 2 in the first position, perpendicular to the plane of the Sp pair.

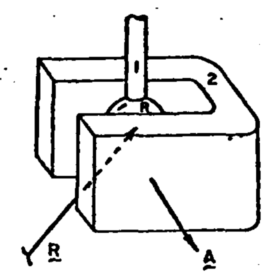

Fig. 3. Sphere-plane (S_p) pair.

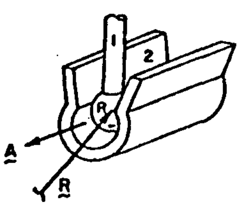

Fig. 4. Sphere-groove (S_g) pair.

†See Appendix for the derivation from the complete constraint equation.

After some displacement of the mechanism, these vectors, in general, will separate due to the relative motion of the joint elements. Noting that both bodies 1 and 2 are connected to ground by C pairs, we use the equations developed by Kohli and Soni[26] for expressing the direction of a vector embedded in the rigid body and also the displaced position of a point of the body after a rotation θ about the cylinder axis and a transition S along the same axis. Using the prime notation for positions of the vector $\mathbf{A}'$ obtained from the motion of body 2 and the unprimed notation for positions of vector $\mathbf{A}$ (assumed frozen in body 1 in the first position and then moving with body 1) from the motion of body 1, the displaced directions of the vector $\mathbf{A}$ in bodies 1 and 2 are

$$\mathbf{A}_j = \cos\theta_{Aj}[\mathbf{A} - (\mathbf{A}\cdot\mathbf{u}_A)\mathbf{u}_A] + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{A}) + (\mathbf{A}\cdot\mathbf{u}_A)\mathbf{u}_A \tag{3}$$

$$\mathbf{A}_j' = \cos\theta_{Bj}[\mathbf{A} - (\mathbf{A}\cdot\mathbf{u}_B)\mathbf{u}_B] + \sin\theta_{Bj}(\mathbf{u}_B + \mathbf{A}) + (\mathbf{A}\cdot\mathbf{u}_B)\mathbf{u}_B \tag{4}$$

Also, the displaced position of the point R in rigid bodies 1 and, 2 are given by:

$$\mathbf{R}_j = \cos\theta_{Aj}[(\mathbf{R}-\mathbf{P}) - ((\mathbf{R}-\mathbf{P})\cdot\mathbf{u}_A]] + \sin\theta_{Aj} \times (\mathbf{u}_A \times (\mathbf{R}-\mathbf{P})) + [(\mathbf{R}-\mathbf{P})\cdot\mathbf{u}_A]\mathbf{u}_A + \mathbf{u}_A S_{Aj} + \mathbf{P} \tag{5}$$

$$\mathbf{R}_j' = \cos\theta_{Bj}[(\mathbf{R}-\mathbf{Q}) - ((\mathbf{R}-\mathbf{Q})\cdot\mathbf{u}_B)\mathbf{u}_B] + \sin\theta_{Bj} \times (\mathbf{u}_{Bj}(\mathbf{u}_B \times (\mathbf{R}-\mathbf{Q})) + [(\mathbf{R}-\mathbf{Q})\cdot\mathbf{u}_B]\mathbf{u}_B + \mathbf{u}_B S_{Bj} + \mathbf{Q}. \tag{6}$$

Using the identity $[\mathbf{A} - (\mathbf{A}\cdot\mathbf{u}_A)\mathbf{u}_A] = (\mathbf{u}_A \times \mathbf{A}) \times \mathbf{u}_A$, introducing the vectors

$$\mathbf{K} = \mathbf{R} - \mathbf{P}$$
$$\mathbf{L} = \mathbf{R} - \mathbf{Q} \tag{7}$$

and the following notation for any two vectors $\mathbf{u}_c$ and $\mathbf{D}$,

$$\mathbf{U}_{CD} = (\mathbf{u}_C \times \mathbf{D}) \times \mathbf{u}_C, \tag{7a}$$

we can substitute eqns (7) and (7a) into eqns (5) and (6) to get

$$\mathbf{R}_j = \mathbf{R} + \mathbf{u}_A S_A + (\cos\theta_{Aj} - 1)\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K}) \tag{5a}$$

and

$$\mathbf{R}_j' = \mathbf{R} + \mathbf{u}_B S_B + (\cos\theta_{Bj} - 1)\mathbf{U}_{BL} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{L}). \tag{6a}$$

We now take the time-derivatives of equations for $\mathbf{R}_j$ and $\mathbf{R}_j'$ and using the notation of dots above the variables to indicate time derivatives, we obtain the following equations

$$\dot{\mathbf{R}}_j = \mathbf{u}_A \dot{S}_{Aj} + [\cos\theta_{Aj}(\mathbf{u}_A \times \mathbf{K}) - \sin\theta_{Aj}\mathbf{U}_{AK}]\dot{\theta}_{Aj} \quad (8)$$

$$\dot{\mathbf{R}}'_j = \mathbf{u}_B \dot{S}_{Bj} + [\cos\theta_{Bj}(\mathbf{u}_B \times \mathbf{L}) - \sin\theta_{Bj}\mathbf{u}_{BL}]\dot{\theta}_{Bj} \quad (9)$$

$$\ddot{\mathbf{R}}_j = \mathbf{u}_A \ddot{S}_{Aj} - [\cos\theta_{Aj}\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K})]\dot{\theta}^2_{Aj} + [\cos\theta_{Aj}(\mathbf{u}_A \times \mathbf{K}) - \sin\theta_{Aj}\mathbf{u}_{AK}]\ddot{\theta}_{Aj} \quad (10)$$

$$\ddot{\mathbf{R}}'_j = \mathbf{u}_B \ddot{S}_{Bj} - [\cos\theta_{Bj}U_{BL} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{L})]\dot{\theta}^2_{Bj} + [\cos\theta_{Bj}(\mathbf{u}_B \times \mathbf{L}) - \sin\theta_{Bj}\mathbf{U}_{BL}]\ddot{\theta}_{Bj}. \quad (11)$$

substituting eqn (7a) into eqn (4), using eqns (5a) and (6a), and by making the following substitutions

$$\mathbf{M}_{Aj} = \cos\theta_{Aj}(\mathbf{u}_A \times \mathbf{K}) - \sin\theta_{Aj}\mathbf{U}_{AK}$$
$$\mathbf{M}_{Bj} = \cos\theta_{Bj}(\mathbf{u}_B \times \mathbf{L}) - \sin\theta_{Bj}\mathbf{U}_{BL} \quad (12)$$

$$\mathbf{N}_{Aj} = \cos\theta_{Aj}\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K})$$
$$\mathbf{N}_{Bj} = \cos\theta_{Bj}\mathbf{U}_{BL} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{L}), \quad (13)$$

we can derive the following *working equations*

$$\mathbf{A}'_j = \mathbf{A} + (\cos\theta_{Bj} - 1)\mathbf{U}_{BA} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{A}) \quad (14)$$

$$\dot{\mathbf{A}}'_j = [\cos\theta_{Bj}(\mathbf{u}_B \times \mathbf{A}) - \sin\theta_{Bj}\mathbf{U}_{BA}]\dot{\theta}_{Bj} = \mathbf{V}_{Bj}\dot{\theta}_{Bj} \quad (15)$$

$$\ddot{\mathbf{A}}'_j = [\cos\theta_{Bj}(\mathbf{u}_B \times \mathbf{A}) - \sin\theta_{Bj}\mathbf{U}_{BA}]\ddot{\theta}_{Bj} - [\cos\theta_{Bj}\mathbf{U}_{BA} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{A})]\dot{\theta}^2_{Bj}$$

$$\ddot{\mathbf{A}}'_j = \mathbf{V}_{Bj}\ddot{\theta}_{Bj} - \mathbf{W}_{Bj}\dot{\theta}^2_{Bj} \quad (16)$$

where

$$\mathbf{V}_{Bj} = \cos\theta_{Bj}(\mathbf{u}_B \times \mathbf{A}) - \sin\theta_{Bj}\mathbf{U}_{BA}$$

and

$$\mathbf{W}_{Bj} = \cos\theta_{Bj}\mathbf{U}_{BA} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{A})$$

$$\mathbf{R}_j - \mathbf{R}'_j = \mathbf{u}_A S_{Aj} + (\cos\theta_{Aj} - 1)\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K}) - \mathbf{u}_B S_{Bj} - (\cos\theta_{Bj} - 1)\mathbf{U}_{BL} - \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{L}) \quad (17)$$

$$\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j = \mathbf{u}_A \dot{S}_{Aj} + \mathbf{M}_{Aj}\dot{\theta}_{Aj} - \mathbf{u}_B \dot{S}_{Bj} - \mathbf{M}_{Bj}\dot{\theta}_{Bj} \quad (18)$$

$$\ddot{\mathbf{R}}_j - \ddot{\mathbf{R}}'_j = \mathbf{u}_A \ddot{S}_{Aj} - \mathbf{N}_{Aj}\dot{\theta}^2_{Aj} + \mathbf{M}_{Aj}\ddot{\theta}_{Aj} - \mathbf{u}_B \ddot{S}_{Bj} + \mathbf{N}_{Bj}\dot{\theta}^2_{Bj} - \mathbf{M}_{Bj}\ddot{\theta}_{Bj} \quad (19)$$

5. DISPLACEMENT ANALYSIS

To analyse the displacements of a particular 3-link one-degree-of-freedom mechanism containing either the *Sp* or *Sg* pair, we need only to take working eqn (17), apply the constraints of the particular grounded pairs and then substitute the results into the following pair geometry constraint equations for displacements.

For the *Sp* pair,

$$(\mathbf{R}_j - \mathbf{R}'_j) \cdot \mathbf{A}'_j = 0. \quad (20)$$

For the *Sg* pair,

$$(\mathbf{R}_j - \mathbf{R}'_j) = \mathbf{A}'_j S_{Gj}. \quad (21)$$

Observe that eqns (20) and (21) are eqns (1) and (2) with $n = 0$.

The cylindrical pairs used in the derivation may be forced to work as prismatic (P) pairs by letting $\theta \equiv 0$ or may be forced to work as revolute (R) pairs by letting $S \equiv 0$.

5.1 *The* P–Sp–P *case*

For this mechanism, we use $\theta_A \equiv \theta_B \equiv 0$ and eqns (14) and (17) are simplified to

$$\mathbf{R}_j - \mathbf{R}'_j = \mathbf{u}_A S_{Aj} - \mathbf{u}_B S_{Bj}$$

and

$$\mathbf{A}'_j = \mathbf{A}.$$

Substituting in eqn (20), we get

$$(\mathbf{u}_A S_{Aj} - \mathbf{u}_B S_{Bj}) \cdot \mathbf{A} = 0 \quad (22)$$

which simplifies to the input/output equation

$$S_{Bj} = \frac{\mathbf{u}_A \cdot \mathbf{A}}{\mathbf{u}_B \cdot \mathbf{A}} S_{Aj}. \quad (23)$$

5.2 *The* R–Sp–P *case*

θ_A is the input; S_B is the output and $\theta_B \equiv S_A \equiv 0$. Equations (14) and (17) with $\theta_B \equiv S_A \equiv 0$ substituted in eqn (20) provide,

$$[-\mathbf{u}_B S_{Bj} + (\cos\theta_{Aj} - 1)\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K})] \cdot \mathbf{A} = 0.$$

After simplification we obtain

$$S_{Bj} = \frac{[(\cos\theta_{Aj} - 1)\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K})] \cdot \mathbf{A}}{\mathbf{u}_B \cdot \mathbf{A}}. \quad (24)$$

5.3 *The* R–Sp–R *case*

We have for this case $S_A \equiv S_B \equiv 0$, and eqn (14) and (17) are simplified to obtain

$$\mathbf{R}_j - \mathbf{R}'_j = (\cos\theta_{Aj} - 1)\mathbf{U}_{AK} + \sin\theta_{Aj}(\mathbf{u}_A \times \mathbf{K}) - (\cos\theta_{Bj} - 1)\mathbf{U}_{BL} - \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{L})$$

and

$$\mathbf{A}'_j = \mathbf{A} + (\cos\theta_{Bj} - 1)\mathbf{U}_{BA} + \sin\theta_{Bj}(\mathbf{u}_B \times \mathbf{A}).$$

Substituting the above equations into eqn (20); and simplifying the resulting equation, we obtain

$$-\mathbf{S}_j \cdot \mathbf{A} + (\cos\theta_{Bj} - 1)[-\mathbf{U}_{BL} \cdot \mathbf{A} - \mathbf{S}_j \cdot \mathbf{U}_{BA}] + \sin\theta_{Bj}[(\mathbf{u}_B \times \mathbf{L}) \cdot \mathbf{A} - \mathbf{S}_j \cdot (\mathbf{u}_B \times \mathbf{A})] = 0 \quad (25)$$

where S_j is the known vector

$$S_j = (\cos\theta_{Aj} - 1)U_{AK} + \sin\theta_{Aj}(u_A \times K). \quad (26)$$

Equation (25) can be solved for θ_{Bj} by using the following identities

$$\cos\theta_B = \frac{1 - \tan^2\frac{\theta_B}{2}}{1 + \tan^2\frac{\theta_B}{2}}; \quad \sin\theta_B = \frac{2\tan\frac{\theta_B}{2}}{1 + \tan^2\frac{\theta_B}{2}} \quad (27)$$

and simplifying the resulting quadratic equation to yield

$$\tan\frac{\theta_B}{2}j = \frac{-b \pm \sqrt{(b^2 - c(c - 2a))}}{c - 2a} \quad (28)$$

where:

$$a = -U_{BL} \cdot A - S_j \cdot U_{BA}$$
$$b = (u_B \times L) \cdot A - S_j \cdot (u_B \times A)$$
$$c = -S_j \cdot A.$$

5.4 *The* P–Sp–R *case*

Here, $\theta_A \equiv S_B \equiv 0$ and we have

$$R_j = R_j' = u_A S_{Aj} - (\cos\theta_{Bj} -)U_{BL} - \sin\theta_{Bj}(u_B \times L)$$

and

$$A_j' = A + (\cos\theta_{Bj} - 1)U_{BA} + \sin\theta_{Bj}(u_B \times L).$$

Substituting the equations above into eqn (20) and simplifying, we get

$$(\cos\theta_{Bj} - 1)[-U_{BL} \cdot A - S_{Aj}(u_A \cdot U_{BA})] + \sin\theta_{Bj}$$
$$[(u_B \times L) \cdot A - S_{Aj}u_A \cdot (u_B \times A)] - S_{Aj}u_A \cdot A = 0. \quad (29)$$

Substituting eqns (27) in eqn (29) and simplifying the resulting quadratic gives us

$$\tan\frac{\theta_B}{2}j = \frac{-b \pm \sqrt{(b_2 - c(c - 2a)}}{c - 2a} \quad (30)$$

where this time

$$a = -U_{BL} \cdot A - S_{Aj}(u_A \cdot U_{BA})$$
$$S = (u_B \times L) \cdot A - S_{Aj}u_A \cdot (u_B \times A)$$
$$c = -]_{Aj}u_A \cdot A.$$

5.5 *The* R–Sg–C *case*

Only S_4 in eqn (17) is identically zero, so we get

$$R_j - R_j' = -u_B S_{Bj} + S_j - (\cos\theta_{Bj} - 1)U_{BL}$$
$$- \sin\theta_{Bj}(u_B \times L)$$

where S_j is given by eqn (26). Also,

$$A_j' = A + (\cos\theta_{Bj} - 1)U_{BA} + \sin\theta_{Bj}(u_B \times A)$$

Substituting in eqn (21), we have

$$u_B S_{Bj} - S_j + (\cos\theta_{Bj} - 1)U_{BL}$$
$$+ \sin\theta_{Bj}(u_B \times L) + A_j' S_{Gj} = 0.$$

Taking the dot product of eqn (31) with $(A_j' \times u_B)$ and upon simplification, we get

$$\cos\theta_{Bj}[S_j \cdot (A \times u_B) + U_{BL} \cdot (A \times u_B)] + \sin\theta_{Bj}$$
$$\times [S_j \cdot U_{BA} + U_{BL} \cdot U_{BA}] - (u_B \times L) \cdot U_{BA} = 0. \quad (32)$$

Again, θ_{Bj} can be obtained by substituting eqns (27) into eqn (32) to obtain a quadratic whose solutions are

$$\tan\frac{\theta_B}{2}j = \frac{-b \pm \sqrt{(a^2 + b^2 - c_2)}}{c - a} \quad (33)$$

where

$$a = S_j \cdot (A \times u_B) + U_{BL} \cdot (A \times u_B)$$
$$b = S_j \cdot U_{BA} + U_{BL} \cdot U_{BA}$$
$$c = -(u_B \times L) \cdot U_{BA}.$$

Taking the dot product of eqn (31) with $(u_B \times L)$ and simplifying, we get

$$S_{Gj} = \frac{[S_j \cdot (u_B \times L) - (\cos\theta_{Bj} - 1)U_{BL} \cdot (u_B \times L)]}{A_j' \cdot (u_B \times L)}$$
$$- \frac{\sin\theta_{Bj}(u_B \times L) \cdot (u_B \times L)}{A_j' \cdot (u_B \times L)}. \quad (34)$$

Taking the dot product of eqn (31) with u_B and simplifying, we get

$$S_{Bj} = [S_j - (\cos\theta_{Bj} - 1)U_{BL} - \sin\theta_{Bj}(u_B \times L)$$
$$- S_{Gj}A_j'] \cdot u_B. \quad (35)$$

6. VELOCITY AND ACCELERATION ANALYSIS

To obtain the velocity and acceleration relations, we can either (a) take the derivatives with respect to time of the displacement equations or (b) use the higher order constraint equations. For the *P–Sp–P* case, taking the derivative of the displacement equation is trivial. But for the other cases, this procedure is cumbersome. It is therefore more convenient to just use eqns (14)–(19) in the following constraint eqns (36)–(39), which are eqns (1) and (2) with $n = 1$ and $n = 2$.

For the *Sp* pair

$$(\dot{R}_j - \dot{R}_j') \cdot A_j' + (R_j - R_j') \cdot \dot{A}_j' = 0 \quad (36)$$

and

$$(\ddot{R}_j - \ddot{R}_j') \cdot A_j' + 2(\dot{R}_j - \dot{R}_j') \cdot \dot{A}_j' + (R_j - R_j') \cdot \ddot{A}_j' = 0. \quad (37)$$

For the S_g pair

$$\dot{R}_j - \dot{R}'_j = A'_j \dot{S}_{Gj} + \dot{A}'_j S_{Gj} \tag{38}$$

and

$$\ddot{R}_j - \ddot{R}'_j = \ddot{A}'_j S_{Gj} + 2\dot{A}'_j \dot{S}_{Gj} + A'_j \ddot{S}_{Gj} \tag{39}$$

6.1 *The* P–Sp–P *case*

Here we can use the time derivatives of the displacement equation to get

$$S_{Bj} = \frac{u_A \cdot A}{u_B \cdot A} S_{Aj}$$

$$\dot{S}_{Bj} = \frac{u_A \cdot A}{u_B \cdot A} \dot{S}_{Aj} \tag{40}$$

$$\ddot{S}_{Bj} = \frac{u_A \cdot A}{u_B \cdot A} \ddot{S}_{Aj}. \tag{41}$$

6.2 *The* R–Sp–P *case*

Equations (18) and (19) become

$$\dot{R}_j - \dot{R}'_j = M_{Aj} \dot{\theta}_{Aj} - u_B \dot{S}_{Bj}$$

and

$$\ddot{R}_j - \ddot{R}'_j = -N_{Aj} \dot{\theta}^2_{Aj} + M_{Aj} \ddot{\theta}_{Aj} - u_B \ddot{S}_{Bj}$$

also

$$A'_j = A_j,\ \dot{A}'_j = \ddot{A}'_j = 0.$$

Substituting in eqns (36) and (37), we get

$$\dot{S}_{Bj} = \frac{M_{Aj} \cdot A}{u_B \cdot A} \dot{\theta}_{Aj} \tag{42}$$

and

$$\ddot{S}_{Bj} = -\frac{N_{Aj} \cdot A}{u \cdot A} \dot{\theta}^2_{Aj} + \frac{M_{Aj} \cdot A}{u_B \cdot A} \ddot{\theta}_{Aj}. \tag{43}$$

6.3 *The* R–Sp–R *case*

$$S_{Aj} \equiv S_{Bj} \equiv \dot{S}_{Aj} \equiv \dot{S}_{Bj} \equiv \ddot{S}_{Aj} \equiv \ddot{S}_{Bj} \equiv 0.$$

Equations (18) and (9) become

$$\dot{R}_j - \dot{R}'_j = M_{Aj} \dot{\theta}_{Aj} - M_{Bj} \dot{\theta}_{Bj}$$

and

$$\ddot{R}_j - \ddot{R}'_j = -N_{Aj} \dot{\theta}^2_{Aj} + M_{Aj} \ddot{\theta}_{Aj} + N_{Bj} \dot{\theta}^2_{Bj} M_{Bj} \ddot{\theta}_{Bj}.$$

Also,

$$\dot{A}'_j = V_{Bj} \dot{\theta}_{Bj};\ \ddot{A}'_j = V_{Bj} \ddot{\theta}_{Bj} - W_{Bj} \dot{\theta}^2_{Bj}.$$

Substituting in eqns (36) and (37), we get

$$\dot{\theta}_{Bj} = \frac{M_{Aj} \cdot A'_j}{M_{Bj} \cdot A'_j - (R_j - R'_j) \cdot V_{Bj}} \dot{\theta}_{Aj} \tag{44}$$

and

$$\ddot{\theta}_{Bj} = \frac{-N_{Aj} \cdot A'_j}{D} \dot{\theta}^2_{Aj} + \frac{M_{Aj} \cdot A'_j}{D} \ddot{\theta}_{Aj} + \frac{2M_{Aj} \cdot V_{Bj}}{D} \dot{\theta}_{Aj} \dot{\theta}_{Bj} + \frac{N_{Bj} \cdot A'_j - 2M_{Bj} \cdot 2M_{Bj} \cdot V_{Bj} - (R_j - R'_j) \cdot W_{Bj}}{D} \dot{\theta}^2_{Bj} \tag{45}$$

where

$$D = M_{Bj} \cdot A'_j - (R_j - R'_j) \cdot V_{Bj}. \tag{46}$$

6.4 *The* P–Sp–R *case*

Equations (18) and (19) are

$$\dot{R}_j - \dot{R}'_j = u_A \dot{S}_{Aj} - M_{Bj} \dot{\theta}_{Bj}$$

$$\ddot{R}_j - \ddot{R}'_j = u_A \ddot{S}_{Aj} + N_{Bj} \dot{\theta}^2_{Bj} - M_{Bj} \ddot{\theta}_{Bj}$$

also,

$$\dot{A}'_j = V_{Bj} \dot{\theta}_{Bj} \text{ and } \ddot{A}'_j = V_{Bj} \ddot{\theta}_{Bj} - W_{Bj} \dot{\theta}^2_{Bj}.$$

Substituting in eqns (36) and (37) we get

$$\dot{\theta}_{Bj} = \frac{u_A \cdot A'_j}{M_{Bj} \cdot A'_j - (R_j - R'_j) \cdot V_{Bj}} \dot{S}_{Aj} \tag{47}$$

and

$$\ddot{\theta}_{Bj} = \frac{1}{D} [u_A \ddot{S}_{Aj} + 2(u_A \cdot V_{Bj}) \dot{S}_A \dot{\theta}_{Bj} + (N_{Bj} \cdot A'_j - 2M_{Bj} \cdot V_{Bj} - (R_j - R'_j) \cdot W_{Bj}) \dot{\theta}^2_{Bj}] \tag{48}$$

where D is given by eqn (46).

6.5 *The* R–Sg–C *case*

Only S_{Aj}, $\dot{S}_{Aj}$ and $\ddot{S}_{Aj}$ are zero and eqns (18) and (19) become:

$$\dot{R}_j - \dot{R}'_j = M_{Aj} \dot{\theta}_{Aj} - u_B \dot{S}_{Bj} - M_{Bj} \dot{\theta}_{Bj}$$

and

$$\ddot{R}_j - \ddot{R}'_j = -N_{Aj} \dot{\theta}^2_{Aj} + M_{Aj} \ddot{\theta}_{Aj} - u_B \ddot{S}_{Bj} + N_{Bj} \dot{\theta}^2_{Bj} - M_{Bj} \ddot{\theta}_{Bj}$$

also,

$$\dot{A}'_j = V_{Bj} \dot{\theta}_{Bj} \text{ and } \ddot{A}'_j = V_{Bj} \ddot{\theta}_{Bj} - W_{Bj} \dot{\theta}^2_{Bj}.$$

Substituting the expression for $(\dot{R}_j - \dot{R}'_j)$ just obtained into eqn (38) we get

$$M_{Aj} \dot{\theta}_{Aj} - u_B \dot{S}_{Bj} - M_{Bj} \dot{\theta}_{Bj} = A'_j \dot{S}_{Gj} + V_{Bj} \dot{\theta}_{Bj} S_{Gj} \tag{49}$$

$\dot{\theta}_{Bj}$, $\dot{S}_{Bj}$ and $\dot{S}_{Bj}$ are unknowns in eqn (49).

Taking the dot product of eqn (49) with $(A'_j \times u_B)$, we get

$$(M_{Aj} \dot{\theta}_{Aj} - M_{Bj} \dot{\theta}_{Bj}) \cdot (A'_j \times u_B) = V_{Bj} \cdot (A'_j \times u_B) \dot{\theta}_{Bj} S_{Gj}$$

or

$$\dot{\theta}_{Bj} = \frac{M_{Aj} \cdot A'_j \times u_B}{(S_{Gj} V_{Bj} + M_{Bj}) \cdot A'_j \times u_B}. \tag{50}$$

Now, taking the dot product of eqn (49) with $[A'_j \times (S_{Gj} V_{Bj} + M_{Bj})]$, we have

$$(M_{Aj} \dot{\theta}_{Aj} - u_B \dot{S}_{Bj}) \cdot A'_j \times (S_{Gj} V_{Bj} + M_{Bj}) = 0$$

or

$$\dot{S}_{Bj} = \frac{M_{Aj}\cdot A_j' \times (S_{Gj}V_{Bj} + M_{Bj})}{u_B \cdot A_j' \times (S_{Gj}V_{Bj} + M_{Bj})}\dot{\theta}_{Aj} \quad (51)$$

Again taking the dot product of eqn (49) with

$$u_B \times (S_{Gj}V_{Bj} + M_{Bj}),$$

we have

$$(M_A\dot{\theta}_{Aj} - A_j'\dot{S}_{Gj})\cdot u_B \times (S_{Gj}V_{Bj} + M_{Bj}) = 0$$

or

$$\dot{S}_{Gj} = \frac{M_{Aj}\cdot u_B \times (S_{Gj}V_{Bj} + M_{Bj})}{A_j' \cdot u_B \times (S_{Gj}V_{Bj} + M_{Bj})}\dot{\theta}_{Aj} \quad (52)$$

Acceleration: Substituting the expression for $(\ddot{R}_j - \ddot{R}_j')$ obtained earlier for the *R–Sg–C* case into eqn (39), we will get

$$\begin{aligned} -N_A\dot{\theta}_{Aj}^2 + M_A\ddot{\theta}_{Aj} - u_B\ddot{S}_{Bj} + N_B\dot{\theta}_{Bj}^2 - M_B\ddot{\theta}_{Bj} \\ = (V_B\ddot{\theta}_{Bj} - W_B\dot{\theta}_{Bj}^2)S_{Gj} + 2V_B\dot{\theta}_{Bj}\dot{S}_{Gj} + A_j'\ddot{S}_{Gj} \end{aligned}$$

or

$$\begin{aligned} u_B\ddot{S}_{Bj} + A_j'\ddot{S}_{Gj} + (S_{Gj}V_{Bj} + M_{Bj})\ddot{\theta}_{Bj} \\ = -N_A\dot{\theta}_{Aj}^2 + M_A\ddot{\theta}_{Aj} + (N_{Bj} + S_{Gj}W_{Bj})\dot{\theta}_{Bj}^2 \\ - 2V_B\dot{\theta}_{Bj}\dot{S}_{Gj} \end{aligned} \quad (53)$$

Letting X be equal to the r.h.s. of eqn (53) and by using the same technique of taking the dot product of eqn (53) with the proper cross-products, we will obtain the following

$$\ddot{\theta}_{Bj} = \frac{X\cdot A_j' \times u_B}{(S_{Gj}V_{Bj} + M_{Bj})\cdot A_j' \times u_B} \quad (54)$$

$$\ddot{S}_{Bj} = \frac{X\cdot A_j' \times (S_{Gj}V_{Bj} + M_{Bj})}{u_B \cdot A_j' \times (S_{Gj}V_{Bj} + M_{Bj})} \quad (55)$$

$$\ddot{S}_{Gj} = \frac{X\cdot u_B \times (S_{Gj}V_{Bj} + M_{Bj})}{A_j' \cdot u_B \times (S_{Gj}V_{Bj} + M_{Bj})}. \quad (56)$$

7. NUMERICAL EXAMPLES

1. Analysis of a *R–Sp–R* mechanism.
The vectors describing the mechanism are

$$\begin{aligned} u_A &= 0\hat{i} + 1\hat{j} + 0\hat{k} \\ u_B &= (3\hat{i} + 1\hat{j} + 0\hat{k})1/\sqrt{(10)} \\ P &= 0\hat{i} + 0\hat{j} + 0\hat{k} \\ Q &= 0\hat{i} + 4\hat{j} + 0.75\hat{k} \\ R &= 1\hat{i} + 1.5\hat{j} + 2\hat{k} \\ A &= 0\hat{i} + 0\hat{j} + 1\hat{k} \end{aligned}$$

The plot of the output displacement (θ_B), velocity $(\dot{\theta}_B)$ and acceleration $(\ddot{\theta}_B)$ are given in Fig. 5.

2. Displacement, velocity and acceleration analysis of a *R–Sg–C* mechanism.

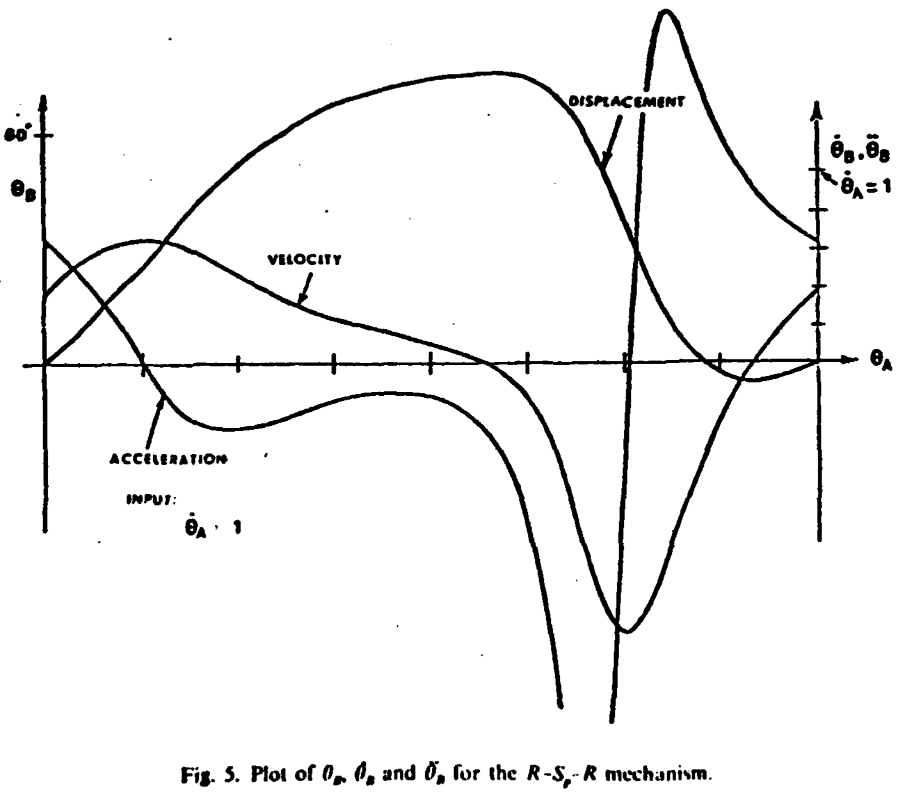

Fig. 5. Plot of θ_B, $\dot{\theta}_B$ and $\ddot{\theta}_B$ for the R-S_p-R mechanism.

Table of displacements, velocities and accelerations

θ_A	θ_B	$\dot\theta_B$	$\ddot\theta_B$	S_G	$\dot S_G$	$\ddot S_C$	S_B	$\dot S_B$	$\ddot S_B$
5	2.12	.41	-.20	-.13	-1.56	-.75	.15	1.76	.36
40	12.99	.12	-1.23	-1.22	-1.95	-.41	1.26	1.79	-.32
80	-95.87	-3.36	32.3	-2.05	1.66	5.58	2.04	-.81	-4.07
120	-112.24	.26	.40	-.64	1.99	-.55	1.11	-1.59	-.43
160	-93.34	.39	.11	.56	1.42	-1.0	-.04	-1.64	.28
200	-81.04	.46	.08	1.32	.68	-1.05	-1.06	-1.20	.90
240	-61.47	.51	.06	1.55	.01	-.84	-1.65	-.44	1.23
280	-40.35	.54	.003	1.37	-0.5	-.67	-1.65	.44	1.24
320	-19.17	.51	-.07	.55	-.97	-.71	-1.05	1.23	.97
355	-2.2	.45	-.16	.13	-1.43	-.77	-.15	1.69	.52

The mechanism parameters are

$$\mathbf{u}_g = (1\hat{i} + 2\hat{j} + 1\hat{k})1/\sqrt{(6)}$$

$$\mathbf{u}_g = (1\hat{i} + \hat{i}\hat{j} + 0\hat{k})1/\sqrt{(2)}$$

$$\mathbf{P} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\mathbf{Q} = 0\hat{i} + 0\hat{j} + 1\hat{k}$$

$$\mathbf{R} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\mathbf{A} = (1\hat{i} + 1\hat{j} + 2\hat{k})1/\sqrt{(6)}.$$

The motion parameters are: $\dot\theta_{Aj}$ is one unit of angular velocity and $\ddot\theta_{Aj}$ is zero, both constant for $j = 0, 1, 2, \ldots$

The results of the analysis for the *R–Sg–C* mechanism are shown in a table on the next page.

The direction of the rotations and linear motions are established using the right hand rule. Rotations are positive counterclockwise looking at the head of the unit vectors $\mathbf{u}_A$ and $\mathbf{u}_B$. Linear motions are positive when they are in the direction of the vectors they are associated with.

It is to be mentioned here also that although the quadratic equations gave two sets of solutions, only one set will define the motion of the mechanism. The other set of solutions are for those positions in which the mechanism has to be disassembled into the other possible configuration.

8. CONCLUSIONS

Displacements, velocities and accelerations have been derived for several three-link spatial mechanisms containing sphere–plane and sphere–groove pairs. The groove of the sphere–groove pair was assumed to be a cylindrical groove, resulting in straight line axis of the groove. However, a more generalized groove may be one whsoe axis is a spatial curve. The authors are working on developing analysis procedures for mechanisms containing such a generalized sphere–groove pair. The expected results of their work will be the subject of a forthcoming paper. Similarly, the authors also have the generalization of the sphere–plane pair in progress, in which the parallel of the pair are generalized to form equidistant curved surfaces.

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APPENDIX

1. *Sphere–plane constraint equation*

The complete displacement contraint equations of the *Sp*-pair are

$$\mathbf{R}_i - \mathbf{R}'_j = S_{rj}\mathbf{u}_{rj} \qquad \text{(a)}$$

and

$$\mathbf{u}'_{rj} \cdot \mathbf{A}'_j = 0 \qquad \text{(b)}$$

where $\mathbf{u}'_{rj}$ is a unit vector in the plane of the *Sp* pair, perpendicular to $\mathbf{A}'_j$ and is in the direction of the relative motion of point R of body 1 with respect to the initially coincident point R' of body 2.

Derivatives of equations (a) and (b) with respect to time are taken to give the following velocity and acceleration constraint equations

Velocity

$$\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j = \dot{S}_{rj}\mathbf{u}'_{rj} + S_{rj}\dot{\mathbf{u}}'_{rj} \qquad \text{(c)}$$

and

$$\dot{\mathbf{u}}'_{rj} \cdot \mathbf{A}'_j + \mathbf{u}'_{rj} \cdot \dot{\mathbf{A}}_j = 0 \qquad \text{(d)}$$

Acceleration:

$$\ddot{\mathbf{R}}_j - \ddot{\mathbf{R}}'_j = \ddot{S}_{rj}\mathbf{u}'_{rj} + 2\dot{S}_{rj}\dot{\mathbf{u}}'_{rj} + S_{rj}\ddot{\mathbf{u}}'_{rj} \qquad \text{(e)}$$

and

$$\ddot{\mathbf{u}}'_{rj} \cdot \mathbf{A}'_j + 2\dot{\mathbf{u}}'_{rj} \cdot \dot{\mathbf{A}}'_j + \mathbf{u}'_{rj} \cdot \ddot{\mathbf{A}}'_j = 0. \qquad \text{(f)}$$

The constraint eqns (a)–(f) are complete in the sense that all of the important variables in the motion of thejoint elements are included. Also, the Coriolis component in the acceleration constraint eqn (f) is evident since $\dot{\mathbf{A}}'_j$ is a function of θ_{ij}.

2. *Proof that* $(d^n/dt^n)[(\mathbf{R}_j - \mathbf{R}'_j) \cdot \mathbf{A}'_j] = 0$ $n = 0, 1, 2$ *satisfies the complete* Sp *pair constraint equation*

Without loss of generality, we can let $\mathbf{S}_{rj} = S_{rj}\mathbf{u}'_{rj}$ and write the complete constraint equation as

$$\frac{d^n}{dt^n}(\mathbf{r}_j - \mathbf{R}'_j) = \frac{d^n}{dt^n}(\mathbf{S}_{rj}) \qquad \text{(a)}$$

and

$$\frac{d^n}{dt^n}(\mathbf{s}_{rj} \cdot \mathbf{A}'_j) = 0. \qquad \text{(b)}$$

Displacement: For $n = 0$, eqn (a) and (b) are

$$(\mathbf{R}_j - \mathbf{R}'_j) = \mathbf{S}_{rj} \qquad \text{(c)}$$

and

$$\mathbf{S}_{rj} \cdot \mathbf{A}'_j = 0. \qquad \text{(d)}$$

Taking the dot product of eqn (c) with $\mathbf{A}'_j$ gives us the displacement constraint equation for the *Sp* pair.

$$(\mathbf{R}_j - \mathbf{R}'_j) \cdot \mathbf{A}'_j = 0. \qquad \text{(e)}$$

Velocity: With $n = 1$, eqns (a) and (b) will become

$$\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j = \dot{\mathbf{S}}_{rj} \qquad \text{(f)}$$

and

$$\dot{\mathbf{S}}_{rj} \cdot \mathbf{A}'_j = -\mathbf{S}_{rj} \cdot \dot{\mathbf{A}}'_j. \qquad \text{(g)}$$

Taking the dot product of eqn (f) with $\mathbf{A}'_j$ gives us

$$(\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j) \cdot \mathbf{A}'_j = \dot{\mathbf{S}}_{rj} \cdot \mathbf{A}'_j. \qquad \text{(h)}$$

Substituting eqn (g) into (h), wil have

$$(\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j) \cdot \mathbf{A}'_j = -S_{rj} \cdot \dot{\mathbf{A}}'_j. \qquad \text{(i)}$$

Equation (c) can now be substituted in eqn (i) to get

$$(\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j) \cdot \mathbf{A}'_j = -(\mathbf{R}_j - \mathbf{R}'_j) \cdot \mathbf{A}'_j$$

or

$$(\dot{\mathbf{R}}_j - \dot{\mathbf{R}}'_j) \cdot \mathbf{A}'_j + (\mathbf{R}_j - \mathbf{R}'_j \cdot \dot{\mathbf{A}}'_j = 0 \qquad \text{(j)}$$

which is really

$$\frac{d}{dt}[(\mathbf{R}_j - \mathbf{R}'_j) \cdot \mathbf{A}'_j] = 0. \qquad \text{(k)}$$

Acceleration: For $n = 2$, eqns (a) and (b) will be

$$\ddot{\mathbf{R}}_j - \ddot{\mathbf{R}}'_j = \ddot{\mathbf{S}}_{rj} \qquad \text{(l)}$$

and

$$\ddot{S}_n \cdot A_j' + 2\dot{S}_n \cdot \dot{A}_j' + S_n \cdot \ddot{A}_j' = 0. \qquad \text{(m)}$$

Taking the dot product of eqn (l) with A_j' and substituting

$$\ddot{S}_n \cdot A_j' = -2\dot{S}_n \cdot \dot{A}_j' - S_n \cdot \ddot{A}_j'$$

from eqn (m), we will get

$$(\ddot{R}_j - \ddot{R}_j') \cdot A_j' = -2\dot{S}_n \cdot \dot{A}_j - S_n \cdot \ddot{A}_j'$$

or

$$(\ddot{R}_j - \ddot{R}_j') \cdot A_j' + 2\dot{S}_n \cdot \dot{A}_j' + S_n \cdot \ddot{A}_j' = 0. \qquad \text{(n)}$$

Substituting eqns (c) and (f) into eqn (n) gives us

$$(\ddot{R}_j - \ddot{R}_j') \cdot A_j' + 2(\dot{R}_j - \dot{R}_j') \cdot \dot{A}_j' + (R_j - R_j') \cdot \ddot{A}_j' = 0$$

which is

$$\frac{d^2}{dt^2}[(R_j - R_j') \cdot A_j'] = 0. \qquad \text{(o)}$$

ANALYSE CINEMATIQUE DES MECANISMES SPATIAUX A TROIS BARRES CONTENANT LES PAIRES SPHERE-PLAN ET SPHERE-RAINURE

G.N. Sandor, D. Kohli, M. Hernandez, Jr., A. Ghosal

Résumé - On considère généralement qu'une paire dans un mécanisme spatial permet un mouvement relatif de vis entre les membres, ou qu'elle restreint le mouvement des éléments qui lui sont reliés.

En employant les contraintes géométriques des paires de sphère-plan et de sphère-rainure cinématiques, les équations pour le déplacement, la vitesse et l'accélération sont dérivées pour les mécanismes avec trois membres R-Sp-R, R-Sp-P, P-Sp-P, P-Sp-R et R-Sr-C (R: révolute; P: prismatique; C: cylindrique; S: sphérique; Sp: sphère-plan; Sr: sphère-rainure). Pour les valeurs connues de la variable d'entrée, les autres variables sont calculées par des formules non-itératives. Le procédé d'analyse est illustré par des exemples numériques.

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KINEMATIC ANALYSIS OF FOUR-LINK SPACE MECHANISMS CONTAINING SPHERE-GROOVE AND SPHERE-SLOTTED-CYLINDER HIGHER PAIRS

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The geometric constraints of two higher pairs, namely sphere-groove and sphere-slotted-cylinder, are derived. Using these pair geometry constraints, input-output relationships are derived for several mechanisms containing sphere-groove and sphere-slotted-cylinder pairs. The input-output equation for the R-Sg-R-R linkage is obtained as a fourth degree polynominal in the half-tangent of the output crank angle. For other cases of mechanisms containing a sphere-groove pair (such as R-Sg-R-P, R-Sg-P-R) the input-output equation is quadratic. The input-output equations for the R-Sc-C-R and R-Sc-R-C are obtained as eighth degree polynomials in the half-tangent of their output angles. For mechanisms with prismatic output containing a sphere-slotted-cylinder pair, the input-output equation is a second degree polynomial in the output translation.

2. INTRODUCTION

Mechanisms containing higher pairs such as cams, sphere-plane, sphere-groove or sphere-in-slotted-cylinder, provide the designer with opportunities for designing mechanisms and machines to satisfy more complex and exact functional requirements than feasible with only lower pair mechanisms. Higher pair mechanisms in general are compact and have fewer links.

In recent years there has been considerable development in tools for kinematic analysis of spatial mechanisms containing lower pairs, but very little has been done in analyzing spatial mechanisms with higher pairs.

Kinematic analysis of space mechanisms was initiated by the significant contribution of Dimentberg [1]. Dimentberg [2,3] demonstrated the use of dual-numbers and screw calculus to obtain closed-form displacement relationships of an RCCC[1] and other four-, five-, six-, and seven-link spatial mechanisms containing revolute, cylindric, prismatic and helical pairs. Denavit [4] derived closed-form displacement relationships for an RCCC mechanism using dual Euler angles. Yang [5] used dual quaternions to get displacement relationships for an RCCC mechanism. Wallace and Freudenstein [6] used a geometric configuration method to obtain displacement analysis of a general RRERR[2] linkage, also called the Tracta coupling.

[1] Revolute, C - cylindric pair
[2] E - planar pair

Vectors were first used by Chace [7,8] to obtain vector equations for position, velocity and acceleration analysis.

Yang [9] used dual numbers to analyze RCRCR five link spatial mechanisms. Soni and Pamidi [10] extended this application of (3×3) matrices with dual elements to obtain closed-form displacement relationships for RRCCR spatial mechanisms. Soni, Dukkipati and Huang [11] also used (3×3) matrices with dual elements to analyze 6 link single loop and two loop spatial mechanisms containing revolute, prismatic and cylindrical pairs. Yuan [12] developed the use of screw co-ordinates by way of which they developed closed-form displacement relationships for all 3R-2C type spatial mechanisms. Duffy [14] has demonstrated the use of spherical trigonometry and dual numbers to obtain closed form input-output relation for four-, five-, and six-link spatial mechanisms. Duffy and Crane [15] also use the same method for the displacement analysis of a general spatial 7-link, 7R mechanism.

Iterative techniques for analysis of spatial mechanisms were developed by Hartenberg and Denavit [16]. Uicker [17] explored in further detail the matrix approach of Hartenberg and Denavit. Soni and Harrisberger [18] used (3×3) matrices with dual elements for an iterative approach to analyze spatial mechanisms.

Finite screws were used by Kohli and Soni [19,20] to conduct displacement analysis of single-loop and two-loop space mechanisms involving R,C,P,H and S pairs. Recently Kohli and Soni [21], and Kohli and Singh [22] used the method of pair geometric constraints and successive screw displacements to conduct analysis of spatial mechanisms containing lower and higher pair. Sandor, Kohli etc. [23] used the above method to conduct displacement, velocity and acceleration analysis of three link spatial mechanisms containing sphere-plane and sphere-groove pairs.

In the present paper, finite screw displacement, expressed in vector form, and pair geometry constraints, also expressed in vector form, are used to derive the displacement equations for four-link spatial mechanisms containing sphere-groove and sphere-in-slotted-cylinder pairs. Although, the analysis of spatial mechanisms containing sphere-groove and sphere-in-slotted- cylinder pairs can be done by modeling these higher pairs as SP^3 and RRP^3, the procedure is made unnecessarily complicated by introducing these hypothetical joints. The use of finite screws and pair geometry constraints avoids this.

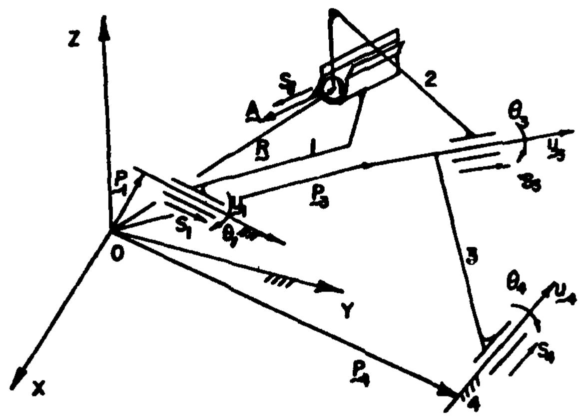


FIGURE 1. THE C-SG-C-C MECHANISM.

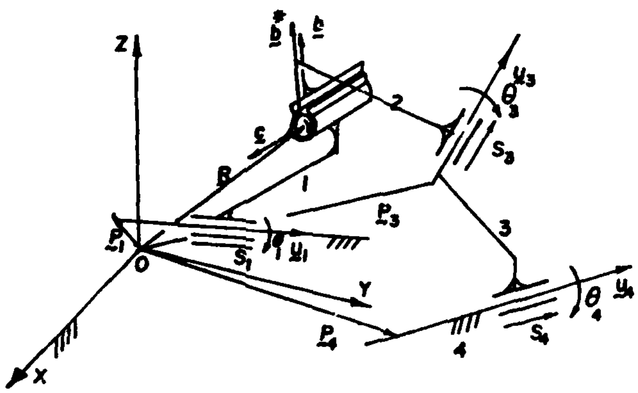


FIGURE 2. THE C-SC-C-C MECHANISMS.

3. THE FOUR-LINK MECHANISMS AND ASSOCIATED VECTORS

Figure 1 shows the C-Sg-C-C mechanism, figure 2 shows the C-Sc-C-C mechanism. Also shown in the figures are the following vectors and scalar quantities. The vectors are denoted by wavy underscores.

$\underset{\sim}{u}_1$ unit vector defining the direction of the first joint axis - grounded cylinder pair 1.

[3] S: spheric pair, P: Prismatic pair, R: Revolute pair.

$\underset{\sim}{u}_3$ unit vector defining the direction of the third joint axis in the initial position - cylinder pair 3.

$\underset{\sim}{u}_4$ unit vector for defining the direction of the fourth joint axis - grounded cylinder pair 4.

$\underset{\sim}{P}_1$ locates the first joint axis $\underset{\sim}{u}_1$.

$\underset{\sim}{P}_3$ locates the third joint axis $\underset{\sim}{u}_3$ in its initial position.

$\underset{\sim}{P}_4$ locates the fourth joint axis $\underset{\sim}{u}_4$.

$\underset{\sim}{A}$ unit vector along the direction of the groove embedded in the groove element.

$\underset{\sim}{b}$ unit vector perpendicular to the axial centerline of the groove defining the orientation of the slot in the initial position, embedded in the groove element.

$\underset{\sim}{c}$ - unit vector along the direction of the groove embedded in the groove element.

$\underset{\sim}{b}^*$ unit vector, coincident with $\underset{\sim}{b}$ in the initial position, embedded in the sphere element.

$\underset{\sim}{c}^*$ unit vector initially coincident with $\underset{\sim}{c}$, embedded in the sphere element.

$\underset{\sim}{A}^*$ unit vector embedded in the sphere element, initially coincident with $\underset{\sim}{A}$

$\underset{\sim}{R}$ vector locating the sphere center in the initial position.

$\underset{\sim}{R}^*$ vector locating a point on the groove axis, but initially coincident with $\underset{\sim}{R}$.

θ_1 rotation of the groove element pivoted at the first C joint.

θ_3 relative rotation of the sphere element pivoted at the third C joint, with respect to link 3 (Fig. 2).

θ_4 rotation of link 3 pivoted at the fourth C joint.

S_1 scalar translation along $\underset{\sim}{u}_1$ at joint 1.

S_3 relative scalar translation of link 2 with respect to link 3 at joint 3.

S_4 scalar translation at joint 4.

S_g relative scalar translation of the sphere element along the groove for Sg pair.

T relative scalar translation of the sphere element along the groove for Sc pair.

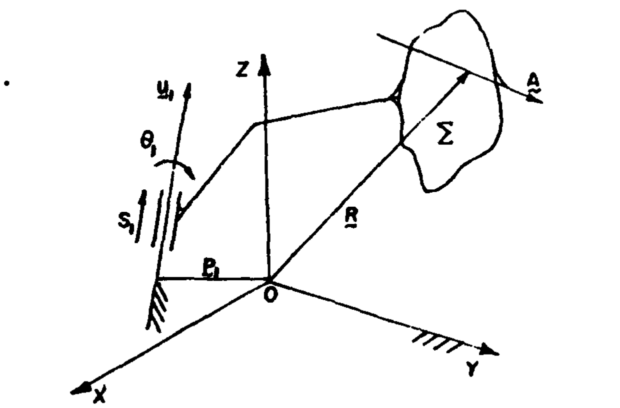


FIGURE 3. VECTOR NOTATION FOR FINITE SCREW DISPLACEMENT.

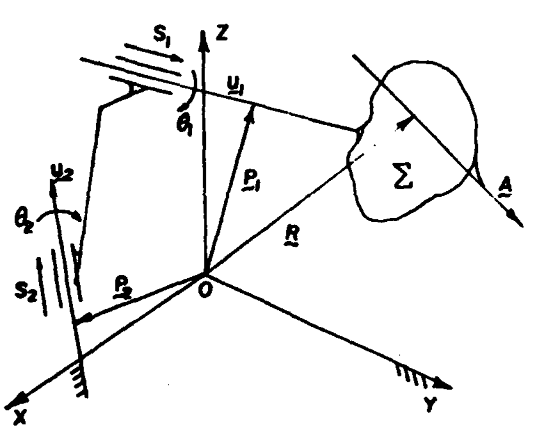


FIGURE 4. VECTOR NOTATION FOR FINITE SUCCESSIVE SCREW DISPLACEMENTS.

4. FINITE SCREW DISPLACEMENTS

Fig. 3 and 4 show a rigid body Σ connected to ground by means of one cylinder pair and by a chain containing two cylinder pairs respectively. By giving successive screw displacements and using a shorthand notation, the displaced position of A and R are obtained as shown in Table I.

<u>Table I</u>

The displaced position of the vector A attached to body Σ in fig 3 is given by

$$A_j = A + (\cos\theta_1 - 1)(u_1 \times A) \times u_1 + \sin\theta_1 (u_1 \times A) \tag{a}$$

For a point R on the vector A, the displaced position is given by

$$R_j = R + (\cos\theta_1 - 1)(u_1 \times (R-P)) \times u_1 + \sin\theta_1 (u_1 \times (R-P)) + S_1 u_1 \tag{b}$$

P is a vector which locates the axis of rotation u_1, and R-P contains the link dimensions, which can be written in terms of constant twist angles and offsets.

The displaced position of A and R in Σ shown in fig 4 (after screw displacement at joints 1 and 2) are obtained as follows.

$$A_j^{21} = A_j^1 + (\cos\theta_2 - 1) U_{2Aj}^{1} + \sin\theta_2 (u_2 \times A_j^1) \tag{c}$$

$$R_j^{21} = R_j^1 + (\cos\theta_2 - 1) U_{2K_2} + \sin\theta_2 (u_2 \times K_2) + u_2 S_2 \tag{d}$$

Table I (continued)

where $A_j = A + (\cos\theta_1 - 1) U_{1A} + \sin\theta_1 (u_1 \times A_1)$ (e)

$$R_j = R + (\cos\theta_1 - 1) U_{1K_1} + \sin\theta_1 (u_1 \times K_1) + u_1 S_1 \tag{f}$$

where,

$$U_{2A_j^1} = (u_2 \times A_j^1) \times u_2 \qquad U_{2K_2} = (u_2 \times K_2) \times u_2$$

$$U_{1A} = (u_1 \times A) \times u_1 \qquad U_{1K_1} = (u_1 \times K_1) \times u_1 \tag{g}$$

$$K_2 = R_j^1 - P_2 \qquad K_1 = R - P_1$$

where u_1 and u_2 are axes of rotation, θ_1, θ_2 are the rotations, S_1 and S_2 are the translations at the joints. P_1 and P_2 locate the axis of rotation u_1 and u_2 in the starting position.

Equations (c), (d), (e), (f) and (g) can be written in a different form:

$$A_j^{21} = A_j^2 + (\cos\theta_1 - 1) X_{1j}^2 + \sin\theta_1 Y_{1j}^2 \tag{h}$$

$$R_j^{21} = R_j^2 + (\cos\theta_1 - 1) C_{1j}^2 + \sin\theta_1 D_{1j}^2 + u_{1j}^2 S_1 \tag{i}$$

where X_{1j}^2, Y_{1j}^2, C_{1j}^2, D_{1j}^2 and u_{1j}^2 are functions of θ_2 and S_2 only and can be expanded like equation (e) and (f)

$$X_{1j}^2 = X_1 + (\cos\theta_2 - 1) U_{2X1} + \sin\theta_2 (u_2 \times X_1)$$

$$Y_{1j}^2 = Y_1 + (\cos\theta_2 - 1) U_{2Y1} + \sin\theta_2 (u_2 \times Y_1)$$

$$C_{1j}^2 = C_1 + (\cos\theta_2 - 1) U_{2C1} + \sin\theta_2 (u_2 \times C_1)$$

$$D_{1j}^2 = D_1 + (\cos\theta_2 - 1) U_{2D1} + \sin\theta_2 (u_2 \times D_1) \tag{j}$$

$$u_{1j}^2 = u_1 + (\cos\theta_2 - 1)[(u_2 \times u_1) \times u_2] + \sin\theta_2 (u_2 \times u_1)$$

$$R_j^2 = R + (\cos\theta_2 - 1)[u_2 \times (R - P_2)) \times u_2] + \sin\theta_2 (u_2 \times (R - P_2)) + u_2 S_2$$

$$A_j^2 = A + (\cos\theta_2 - 1)[(u_2 \times A) \times u_2] + \sin\theta_2 (u_2 \times A)$$

$$U_{2X1} = (u_2 \times X_1) \times u_2$$

$$U_{2Y1} = (u_2 \times Y_1) \times u_2$$

$$U_{2C1} = (u_2 \times C_1) \times u_2$$

$$U_{2D1} = (u_2 \times D_1) \times u_2$$

$$X_1 = (u_1 \times A) \times u_1 = U_{1A}$$

$$Y_1 = (u_1 \times A)$$

$$C_1 = (u_1 \times (R - P_1)) \times u_1 = D_1 \times u_1$$

$$D_1 = (u_1 \times (R - P_1))$$

In the case of a R or P joint, the translation or rotation respectively vanishes and the resulting equations can be considerable simplified.

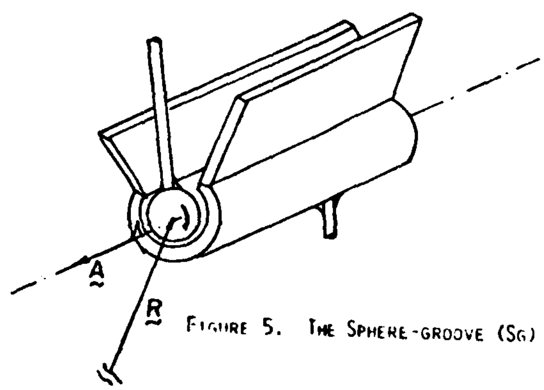


FIGURE 5. THE SPHERE-GROOVE (SG) PAIR.

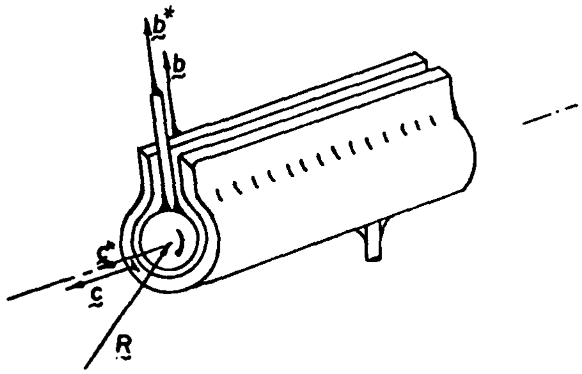


FIGURE 6. THE SPHERE-IN-SLOTTED-CYLINDER (Sc) PAIR.

5. PAIR GEOMETRY CONSTRAINTS FOR THE Sg AND Sc PAIRS

Figure 5 shows the sphere-groove (Sg) pair. The vector $\underline{A}$ defines the axial centerline of the groove and vector $\underline{R}$ locates the sphere-center. Figure 6 shows the sphere-in-slotted-cylinder (Sc) pair. Vector $\underline{c}$ defines the axial centerline of the groove and vector $\underline{b}$ is normal to $\underline{c}$ in the initial position. Vectors $\underline{A}$, $\underline{b}$, and $\underline{c}$ are all unit vectors. Vector $\underline{R}$ locates the sphere center. Vectors $\underline{c}^*$, $\underline{b}^*$ are also shown.

The pair geometry constraints for the sphere-groove and sphere-in-slotted-cylinder pairs can now be defined.

The pair geometry constraint for the Sg pair is given by (see Fig 1)

$$\underline{R}_j^1 - \underline{R}_j^{43} = S_g \underline{A}_j^1 \tag{1}$$

where $\underline{R}_j^1$ is the displaced position of $\underline{R}$ obtained from a screw displacement at joint 1; $\underline{R}_j^{43}$ is the displaced position vector of the sphere center originally located by $\underline{R}$ due to successive screw displacements at joint 3 and 4; and $\underline{A}_j^1$ is the displaced vector $\underline{A}$ due to a screw displacement at joint 1.

The pair geometry constraint for the Sc pair is given by:

$$\underline{R}_j^1 - \underline{R}_j^{43} = \underline{c}_j^1 T \tag{2}$$

Equation (1) and (2) imply that the relative displacement in the higher pair can only be along the groove.

Also, there can be no rotation about the vector $\underline{c}$. This condition can be expressed as

$$(\underline{c}_j^1 \times \underline{b}_j^1) \cdot \underline{b}_j^{*43} = 0 \tag{3}$$

where, $\underline{c}_j^1$ and $\underline{b}_j^1$ are the displaced vectors $\underline{c}$ and $\underline{b}$ due to a screw displacement at joint 1, and $\underline{b}_j^{*43}$ is the displaced vector $\underline{b}^*$ due to successive screw displacements at joints 3 and 4.

The constraint equations for velocity and acceleration can be obtained by taking time derivatives of equations (1), (2), and (3). The general constraint equations for Sg pair can be written as:

$$\frac{d^n}{dt^n}(\underline{R}_j^1 - \underline{R}_j^{43}) = \frac{d^n}{dt^n}(S_g \underline{A}_j^1), \quad n = 0,1,2 \tag{4}$$

and for the Sc pair

$$\frac{d^n}{dt^n}(\underline{R}_j^1 - \underline{R}_j^{43}) = \frac{d^n}{dt^n}(\underline{c}_j^1 T), \quad n = 0,1,2 \tag{5}$$

$$\frac{d^n}{dt^n}[(\underline{c}_j^1 \times \underline{b}_j^1) \cdot \underline{b}_j^{*43}] = 0,1,2 \tag{6}$$

6. ANALYSIS OF MECHANISMS CONTAINING Sg PAIR

To analyze four link mechanisms containing an Sg pair, we need to consider equation (1). The terms can be expanded as in equations (h), (i) and (j) by using proper subscripts and superscripts. In order to get an input-output relation between θ_1 and θ_4 or S_4 (if the fourth joint is prismatic) it is necessary to eliminate θ_3 or S_3 (depending whether the third joint is a revolute or prismatic) and S_g. The S_g pair has four degrees of freedom, so the other joints have to be either P or R joints.

6a. The R-Sg-R-R Case

In this case θ_1 is the input, θ_4 is the output. All the cylinder joints have to be forced to have zero translation to make them revolute joints. Therefore we have the following expressions:

$$\underline{R}_j^1 = \underline{R} + (\cos\theta_1 - 1)[(\underline{u}_1 \times (\underline{R} - \underline{P}_1)) \times \underline{u}_1] + \sin\theta_1 (\underline{u}_1 \times (\underline{R} - \underline{P}_1)) \tag{7}$$

$$\underline{A}_j^1 = \underline{A} + (\cos\theta_1 - 1)[(\underline{u}_1 \times \underline{A}) \times \underline{u}_1] + \sin\theta_1 (\underline{u}_1 \times \underline{A}) \tag{8}$$

$$\underline{R}_j^{43} = \underline{R}_j^4 + (\cos\theta_3 - 1)\underline{C}_{3j}^4 + \sin\theta_3 \underline{D}_{3j}^4 \tag{9}$$

$\underline{R}_j^4$, $\underline{C}_{3j}^4$ and $\underline{D}_{3j}^4$ can be expanded like equation (j) with proper change of subscripts and superscripts. Taking the cross product of equation (1) with $\underline{A}_j^1$ we have

$$\underline{R}_j^1 \times \underline{A}_j^1 = \underline{R}_j^{43} \times \underline{A}_j^1 \tag{10}$$

The left hand side is known since θ_1 is the input. Expanding and then simplifying the right hand side of equation (10) we have,

$$\underline{A}_j^1 \cdot \underline{R}_j^1 = \underline{A}_j^1 \times \underline{R}_j^{43} = \underline{A}_j^1 \times \underline{R}_j^4 + (\cos\theta_3 - 1)(\underline{A}_j^1 \times \underline{C}_{3j}^4) + \sin\theta_3 (\underline{A}_j^1 \times \underline{D}_{3j}^4) \tag{11}$$

Taking the cross product of both sides with $\underline{A}_j^1 \times \underline{D}_{3j}^4$ we have

$$(\cos\theta_3 - 1)(\underline{A}_j^1 \times \underline{C}_{3j}^4) \times (\underline{A}_j^1 \times \underline{D}_{3j}^4) = [\underline{A}_j^1 \times (\underline{R}_j^1 - \underline{R}_j^4)] \times (\underline{A}_j^1 \times \underline{D}_{3j}^4) \tag{12}$$

Simplifying and noting that equation (12) is actually a scalar equation as both vectors are along $\underline{A}_j^1$, we have,

$$(\cos\theta_3-1)[\underset{-}{A}_j^1\cdot(\underset{-}{C}_{3j}^4\times\underset{-}{D}_{3j}^4)] = [\underset{-}{A}_j^1\cdot(\underset{-}{R}_j^1-\underset{-}{R}_j^4)\times\underset{-}{D}_{3j}^4] \tag{13}$$

similarily taking cross product with $\underset{-}{A}_j^1\times\underset{-}{C}_{3j}^4$ and simplifying we have

$$\sin\theta_3[\underset{-}{A}_j^1\cdot(\underset{-}{D}_{3j}^4\times\underset{-}{C}_{3j}^4)] = [\underset{-}{A}_j^1\cdot(\underset{-}{R}_j^1-\underset{-}{R}_j^4)\times\underset{-}{C}_{3j}^4] \tag{14}$$

Equations (13) and (14) can be written as

$$A\cos\theta_3 = B + A \tag{15}$$

$$-A\sin\theta_3 = C \tag{16}$$

$$\text{where } A = \underset{-}{A}_j^1\cdot(\underset{-}{C}_{3j}^4\times\underset{-}{D}_{3j}^4),\ B = \underset{-}{A}_j^1\cdot(\underset{-}{R}_j^1-\underset{-}{R}_j^4)\times\underset{-}{D}_{3j}^4 \qquad C = \underset{-}{A}_j^1\cdot(\underset{-}{R}_j^1-\underset{-}{R}_j^4)\times\underset{-}{C}_{3j}^4 \tag{17}$$

Squaring and adding we have the input-output relationship as,

$$B^2 + 2AB + C^2 = 0 \tag{18}$$

Simplifying expressions in (17) it can be shown that all terms A, B and C are linear in $\sin\theta_4$ and $\cos\theta_4$ (Table II)

<u>Table II</u>

Simplifications of some Vector Expressions:

$$\underset{-}{C}_{3j}^4\times\underset{-}{D}_{3j}^4 = (\underset{-}{C}_3\times\underset{-}{D}_3) + (\cos\theta_4-1)(\underset{-}{C}_3\times((\underset{-}{u}_4\times\underset{-}{D}_3)\times\underset{-}{u}_4) + [(\underset{-}{u}_4\times\underset{-}{C}_3)\times\underset{-}{u}_4)\times\underset{-}{D}_3]$$
$$+\sin\theta_4[-(\underset{-}{X}_2\times\underset{-}{Y}_2)\times\underset{-}{u}_4] + [\underset{-}{X}_2(\underset{-}{Y}_2\times\underset{-}{u}_4)]\underset{-}{u}_4[\sin^2\theta_4 + \cos^2\theta_4 - 2\cos\theta_4 + 1]$$
$$= (\underset{-}{C}_3\times\underset{-}{D}_3) + (\cos\theta_4-1)[-[\underset{-}{u}_4\cdot(\underset{-}{C}_3\times\underset{-}{D}_3)]\underset{-}{u}_4 + (\underset{-}{C}_3\times\underset{-}{D}_3)] + \sin\theta_4[-(\underset{-}{C}_3\times\underset{-}{D}_3)\times\underset{-}{u}_4]$$
$$= (\underset{-}{C}_3\times\underset{-}{D}_3) + (\cos\theta_4-1)[\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))\times\underset{-}{u}_4] = (\underset{-}{C}_3\times\underset{-}{D}_3)_j^4 \tag{a}$$

$$\text{Similarly, } \underset{-}{R}_j^4\times\underset{-}{D}_{3j}^4 = (\underset{-}{k}_4\times\underset{-}{D}_3) + (\cos\theta_4-1)[(\underset{-}{u}_4\times\underset{-}{k}_4\times\underset{-}{D}_3)\times\underset{-}{u}_4]$$
$$+\sin\theta_4(\underset{-}{u}_4\times(\underset{-}{k}_4\times\underset{-}{D}_3)) + \underset{-}{P}_4\times\underset{-}{D}_{3j}^4 \tag{b}$$
$$= (\underset{-}{C}_3\times\underset{-}{D}_3)_j^4 \tag{c}$$

$$\text{Similarly, } \underset{-}{R}_j^4\times\underset{-}{C}_{3j}^4 = (\underset{-}{k}_4\times\underset{-}{C}_3)_j^4 + \underset{-}{P}_4\times\underset{-}{C}_{3j}^4 \tag{d}$$

$$\text{and } \underset{-}{R}_j^4\times\underset{-}{u}_{3j}^4 = (\underset{-}{k}_4\times\underset{-}{u}_4)_j + (\underset{-}{P}_4\times\underset{-}{u}_{3j}^4) \tag{e}$$

$$\underset{-}{X}_{3jb^*}^4 = \underset{-}{X}_{3b^*} + (\cos\theta_4-1)[\underset{-}{u}_4\times\underset{-}{X}_{3b^*})\times\underset{-}{u}_4] + \sin\theta_4(\underset{-}{u}_4\times\underset{-}{X}_{3b^*}) \tag{f}$$

$$\underset{-}{Y}_{3jb^*}^4 = \underset{-}{Y}_{3b^*} + (\cos\theta_4-1)[\underset{-}{u}_4\times\underset{-}{Y}_{3b^*})\times\underset{-}{u}_4] + \sin\theta_4(\underset{-}{u}_4\times\underset{-}{Y}_{3b^*}) \tag{g}$$

$$\underset{-}{X}_{3b^*} = (\underset{-}{u}_3\times\underset{-}{b}^*)\times\underset{-}{u}_3 = \underset{-}{Y}_{3b^*}\times\underset{-}{u}_3 \tag{h}$$

$$\underset{-}{Y}_{3b^*} = (\underset{-}{u}_3\times\underset{-}{b}^*) \tag{i}$$

So the input-output relation is of fourth degree in the tangent of the output half-angle. The input-output relationship is of the form,

$$\alpha_1(\cos\theta_4-1)^2 + \alpha_2\sin^2\theta_4 + \alpha_3\sin\theta_4(\cos\theta_4-1) + \alpha_4(\cos\theta_4-1) + \alpha_5\sin\theta_4 + \alpha_6 = 0 \tag{19}$$

$$\text{using } \cos\theta_4 = \frac{1-x_4^2}{1+x_4^2},\ \sin\theta_4 = \frac{2x_4}{1+x_4^2},\ x_4 = \tan\frac{\theta_4}{2}$$

we get,

$$x_4^2(4\alpha_1-\alpha_2-2\alpha_4) + x_4^3(-4\alpha_3 - 2\alpha_5) + x_4^2(4\alpha_2-2\alpha_4+2\alpha_6) + x_4(2\alpha_5) - \alpha_6 = 0 \tag{20}$$

where α_j, j = 1,2,3,4,5 are defined in Table III.

Table III

$$\alpha_1 = (b_2^2+c_2^2+d_4),\ \alpha_2 = (b_3^2+c_3^2+d_5)$$
$$\alpha_3 = (2b_2b_3+2c_2c_3+d_6),\ \alpha_4 = (2b_1b_2+2c_1c_2+d_2)$$
$$\alpha_5 = (2b_1b_3+2c_1c_3+d_3),\ \alpha_6 = (b_1^2+c_1^2+d_1)$$
$$b_1 = \underset{-}{D}_3\cdot(\underset{-}{A}_j^1\times\underset{-}{R}_j^1) - \underset{-}{A}_j^1\cdot[\underset{-}{K}_4\times\underset{-}{D}_3 + (\underset{-}{P}_4\times\underset{-}{D}_3)]$$
$$b_2 = [(\underset{-}{u}_4\times\underset{-}{D}_3)\times\underset{-}{u}_4]\cdot(\underset{-}{A}_j^1\times\underset{-}{R}_j^1) - \underset{-}{A}_j^1\cdot[(\underset{-}{u}_4\times(\underset{-}{K}_4\times\underset{-}{D}_3))\times\underset{-}{u}_4 - \underset{-}{P}_4\times((\underset{-}{u}_4\times\underset{-}{D}_3)\times\underset{-}{u}_4)]$$
$$b_3 = [(\underset{-}{u}_4\times\underset{-}{D}_3)]\cdot(\underset{-}{A}_j^1\times\underset{-}{R}_j^1) - \underset{-}{A}_j^1\cdot[\underset{-}{u}_4\times(\underset{-}{K}_4\times\underset{-}{D}_3) + \underset{-}{P}_4\times(\underset{-}{u}_4\times\underset{-}{D}_3)]$$
$$c_1 = \underset{-}{C}_3\cdot(\underset{-}{A}_j^1\times\underset{-}{R}_j^1) - \underset{-}{A}_j^1\cdot[(\underset{-}{K}_4\times\underset{-}{C}_3) - (\underset{-}{P}_4\times\underset{-}{C}_3)]$$
$$c_2 = [(\underset{-}{u}_4\times\underset{-}{C}_3)\times\underset{-}{u}_4]\cdot(\underset{-}{A}_j^1\times\underset{-}{R}_j^1) - \underset{-}{A}_j^1\cdot[(\underset{-}{u}_4\times(\underset{-}{K}_4\times\underset{-}{C}_3))\times\underset{-}{u}_4 - \underset{-}{P}_4\times\underset{-}{U}_{4C_3}]$$
$$c_3 = [(\underset{-}{u}_4\times\underset{-}{C}_3)]\cdot[\underset{-}{A}_j^1\times\underset{-}{R}_j^1] - \underset{-}{A}_j^1\cdot[\underset{-}{u}_4\times(\underset{-}{K}_4\times\underset{-}{C}_3)) + \underset{-}{P}_4\times(\underset{-}{u}_4\times\underset{-}{C}_3)]$$
$$d_1 = 2b_1[\underset{-}{A}_j^1\cdot(\underset{-}{C}_3\times\underset{-}{D}_3)]$$
$$d_2 = 2b_1[\underset{-}{A}_j^1\cdot(\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))\times\underset{-}{u}_4] + 2b_2[\underset{-}{A}_j\cdot(\underset{-}{C}_3\times\underset{-}{D}_3)]$$
$$d_3 = 2b_1[\underset{-}{A}_j^1\cdot(\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))] + 2b_3[\underset{-}{A}_j^1\cdot(\underset{-}{C}_3\times\underset{-}{D}_3)]$$
$$d_4 = 2b_2[\underset{-}{A}_j^1\cdot(\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))\times\underset{-}{u}_4]$$
$$d_5 = 2b_3[\underset{-}{A}_j^1\cdot(\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))]$$
$$d_6 = 2b_2[\underset{-}{A}_j^1\cdot(\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))] + 2b_3[\underset{-}{A}_j^1\cdot(\underset{-}{u}_4\times(\underset{-}{C}_3\times\underset{-}{D}_3))\times\underset{-}{u}_4]$$

$$\text{Again using } \cos\theta_3 = \frac{1-x_3^2}{1+x_3^2},\ \sin\theta_3 = \frac{2x_3}{1+x_3^2},$$

$$x_3 = \tan\frac{\theta_3}{2}$$

and using trigonometric identities:

$$\sin\theta_3 x_3 + \cos\theta_3 = 1, \tag{21}$$

$$\sin\theta_3 - x_3\cos\theta_3 = x_3 \tag{22}$$

$$\text{we have, } x_3 = B/C = C/B + 2A \tag{23}$$

using equation (1) and taking the scalar product with $\underset{-}{A}_j^1$ we get,

$$S_g = (\underset{-}{R}_j^1-\underset{-}{R}_j^{43})\cdot\underset{-}{A}_j^1/\underset{-}{A}_j^1\cdot\underset{-}{A}_j^1 \tag{24}$$

Taking the derivatives of equation (20), (23) and (24) we can obtain expression for velocities and accelerations.[4]

6b. <u>The R-Sg-R-P, R-Sg-P-R and R-Sg-P-P Cases.</u>

For mechanisms containing Sg pair, like the R-Sg-R-P mechanism, the R-Sg-P-R mechanism and the R-Sg-P-P mechanism, the input-output relationships can be found after simplifying the expressions for finite screw displacements and the pair geometry constraints (see reference [24] for details).

[4]See also Ref. [24].

For the R-Sg-R-P mechanism the input-output relationship is quadratic in output displacement. For the R-Sg-P-R mechanism, the input-output relationship is quadratic in the output tangent half angle. For the R-Sg-P-P mechanism the input-output relationship is linear.

7. ANALYSIS OF FOUR LINK MECHANISMS CONTAINING AN Sc PAIR

The Sc pair has three degrees of freedom, so there can be only one cylindric joint and the two other joints are revolute or prismatic. To analyze four link mechanisms containing an Sc pair we need to consider equations (2) and (3). In order to get an expression between input θ_1 and output(s) θ_4 and/or S_4, we need to eliminate one joint rotation and/or translation.

7a. The R-Sc-C-R Case

In this case, $S_1 \equiv S_4 \equiv 0$. Equation (2) can be written as,

$$R_j^4 + (\cos\theta_3 - 1)C_{3j}^4 + \sin\theta_3 D_{3j}^4 + u_{3j}^4 S_3 + c_j^1 T = R_j^1 \quad (25)$$

By writing the dot product of equation (25) with c_j^1 and u_{3j}^4 we get after simplification,

$$(1-\cos\theta_3)a_1 + \sin\theta_3 b_1 + c_1 = 0$$

$$\text{where}\quad a_1 = c_j^1 \cdot D_{3j}^4,\quad b_1 = c_j^1 \cdot C_{3j}^4$$
$$c_1 = c_j^1 \cdot (R_j^4 \times u_{3j}^4) - u_{3j}^4 \cdot (c_j^1 \times R_j^1). \quad (26)$$

Simplifying equation (3) we obtain

$$a_2(1-\cos\theta_3) + b_2 \sin\theta_3 + c_2 = 0, \quad (27)$$

$$\text{where}\quad a_2 = -(c_j^1 \times b_j^1) \cdot X_{3jb*}^4$$
$$b_2 = (c_j^1 \times b_j^1) \cdot Y_{3jb*}^4 \quad (28)$$
$$c_2 = (c_j^1 \times b_j^1) \cdot b_j^{*4}$$

In Table II the expanded terms for X_{3jb*}^4, Y_{3jb*}^4 are given. We can eliminate θ_3 from (26) and (27) after using $\cos\theta_3 = \dfrac{1-x_3^2}{1+x_3^2}$ and $\sin\theta_3 = \dfrac{2x_3}{1+x_3^2}$,

$$x_3 = \tan\theta_3/2$$

The eliminant is given by $|ab||bc| - |ac|^2 = 0$, (29) and the common root is given by $x_3 = -\dfrac{|ac|}{|ab|} = \dfrac{|bc|}{|ac|}$ (30)

$$\text{where}\quad |ab| = (2a_1+c_1)(2b_2) - (2b_1)(2a_2+c_2)$$
$$|bc| = 2b_1c_2 - 2b_2c_1 \quad\text{and} \quad (31)$$
$$|ac| = (2a_1+c_1)c_2 - (2a_2+c_2)c_1 = 2a_1c_2 - 2a_2c_1$$

Equation (29) is the input-output relationship. All the terms a_i, b_i, c_i, $i = 1,2$ are linear in $\sin\theta_4$ and $\cos\theta_4$. Equation (29) can be simplified to give $\sum_{i=0}^{8} \delta_{i+1} x_4^i = 0$, a polynomial in $\tan(\theta_4/2)$ in terms of θ_1 and link parameters (refer to reference [24] and Table IV for details.

Table IV

Coefficients of the 8th degree polynomial for R-Sc-C-R Mechanism.

$a_1 = c_j^1 \cdot D_{3j}^4 = \alpha_1 + \alpha_2(\cos\theta_4 - 1) + \alpha_3 \sin\theta_4$

$a_2 = -(c_j^1 \times b_j^1) \cdot X_{3jb*}^4 = A_1 + A_2(\cos\theta_4 - 1) + A_3 \sin\theta_4$

$b_1 = c_j^1 \cdot C_{3j}^4 = \beta_1 + \beta_2(\cos\theta_4 - 1) + \beta_3 \sin\theta_4$

$b_2 = (c_j^1 \times b_j^1) \cdot Y_{3jb*}^4 = B_1 + B_2(\cos\theta_4 - 1) + B_3 \sin\theta_4$

$c_1 = c_j^1 \cdot (R_j^4 \times u_{3j}^4) - u_{3j}^4 \cdot (c_j^1 \times R_j^1) = \gamma_1 + \gamma_2(\cos\theta_4 - 1) + \gamma_3 \sin\theta_4$

$c_2 = (c_j^1 \times b_j^1) \cdot b_j^* = C_1 + C_2(\cos\theta_4 - 1) + C_3 \sin\theta_4$

$\alpha_1 = c_j^1 \cdot D_3,\ \alpha_2 = c_j^1 \cdot U_4 D_3,\ \alpha_3 = c_j^1 \cdot (u_4 \times D_3)$

$\beta_1 = c_j^1 \cdot C_3,\ \beta_2 = c_j^1 \cdot U_4 C_3,\ \beta_3 = c_j^1 \cdot (u_4 \times C_3)$

$\gamma_1 = c_j^1 \cdot [(R_4 \times u_3) + (P_4 \times u_3)] - u_3 \cdot (c_j^1 \times R_j^1)$

$\gamma_2 = c_j^1 \cdot [u_4 \times (R_4 \times u_3)) \times u_4 + P_4 \times U_4 u_3] - (u_4 \times u_3) \times u_4 \cdot (c_j^1 \times R_j^1)$

$\gamma_3 = c_j^1 \cdot [u_4 \times (R_4 \times u_3)) + P_4 (u_4 \times u_3)] - (u_4 \times u_3) \cdot (c_j^1 \times R_j^1)$

$A_1 = -(c_j^1 \times b_j^1) \cdot X_{3b*},\ A_3 = -(c_j^1 \times b_j^1) \cdot (u_4 \times X_{3b*})$

$A_2 = -(c_j^1 \times b_j^1) \cdot (u_4 \times X_{3b*}) \times u_4$

$B_1 = (c_j^1 \times b_j^1) \cdot Y_{3b*},\ B_2 = (c_j^1 \times b_j^1) \cdot (u_4 \times Y_{3b*}) \times u_4$

$B_3 = (c_j^1 \times b_j^1) \cdot (u_4 \times Y_{3b*})$

$C_1 = (c_j^1 \times b_j^1) \cdot b^*$

$C_2 = (c_j^1 \times b_j^1) \cdot [(u_4 \times b^*) \times u_4]$

$C_3 = (c_j^1 \times b_j^1)\ (u_4 \times b^*)$

$|ab| = X_1 + X_2(\cos\theta_4 - 1) + X_3 \sin\theta_4 + X_4 \sin\theta_4(\cos\theta_4 - 1) + X_5 \sin^2\theta_4 + X_6(\cos\theta_4 - 1)^2$

$|bc| = Y_1 + Y_2(\cos\theta_4 - 1) + Y_3 \sin\theta_4 + Y_4 \sin\theta_4(\cos\theta_4 - 1) + Y_5 \sin^2\theta_4 + Y_6(\cos\theta_4 - 1)^2$

$|ac| = Z_1 + Z_2(\cos\theta_4 - 1) + Z_3 \sin\theta_4 + Z_4 \sin\theta_4(\cos\theta_4 - 1) + Z_5 \sin^2\theta_5 + Z_6(\cos\theta_4 - 1)^2$

$X_1 = 2(2\alpha_1 + \gamma_1)B_1 - 2\beta_1(2A_1 + C_1)$

$X_2 = 2[(2\alpha_2 + \gamma_2)B_1 + B_2(2\alpha_1 + \gamma_1)] - 2[\beta_1(2A_2 + C_2) + \beta_2(2A_1 + C_1)]$

$X_3 = 2[B_1(2\alpha_3 + \gamma_3) + B_3(2\alpha_1 + \gamma_1)] - 2[\beta_3(2A_1 + C_1) + \beta_1(2A_3 + C_3)]$

$X_4 = 2[B_2(2\alpha_3 + \gamma_3) + B_3(2\alpha_2 + \gamma_2)] - 2[\beta_2(2A_3 + C_3) + \beta_3(2A_2 + C_2)]$

$X_5 = 2[B_3(2\alpha_3 + \gamma_3)] - 2[\beta_3(2A_3 + C_3)]$

$X_6 = 2[(2\alpha_2 + \gamma_2)B_2] - 2[\beta_2(2A_2 + C_2)]$

$Y_1 = 2\beta_1 C_1 - 2\gamma_1 B_1$

$Y_2 = 2\beta_1 C_2 - 2\gamma_1 B_2 + 2\beta_2 C_1 - 2B_1\gamma_2$

$Y_3 = 2\beta_1 C_3 - 2\gamma_3 B_1 + 2\beta_3 C_1 - 2B_3\gamma_1$

$Y_4 = 2\beta_3 C_2 - 2\gamma_3 B_2 + 2\beta_2 C_3 - 2B_3\gamma_2$

$Y_5 = 2\beta_3 C_3 - 2\gamma_3 C_3$

$Y_6 = 2\beta_2 C_2 - 2B_2\gamma_2$

$Z_1 = 2\alpha_1 C_1 - 2\gamma_1 A_1$

$Z_2 = 2\alpha_1 C_2 + 2\alpha_2 C_1 - 2A_1\gamma_2 - 2A_2\gamma_1$

$Z_3 = 2\alpha_3 C_1 + 2\alpha_1 C_3 - 2A_1\gamma_3 - 2A_3\gamma_1$

$Z_4 = 2\alpha_2 C_3 + 2\alpha_3 C_2 - 2A_2\gamma_3 - 2A_3\gamma_2$

$Z_5 = 2\alpha_3 C_3 - 2A_3\gamma_3$

$Z_6 = 2\alpha_2 C_2 - 2A_2\gamma_2$

Table IV (continued)

$$|ab|\,|bc| - |ac|^2 = 0$$

$$\delta_1 + \delta_2(\cos\theta_4 - 1) + \delta_3\sin\theta_4 + \delta_4\sin\theta_4(\cos\theta_4 - 1) + \delta_5\sin^2\theta_4 + \delta_6(\cos\theta_4 - 1)^2$$
$$+ \delta_5\sin^2\theta_4 + \delta_6(\cos\theta_4 - 1)^2 + \delta_7\sin\theta_4(\cos\theta_4 - 1)^2 + \delta_8(\cos\theta_4 - 1)\sin^2\theta_4$$
$$+ \delta_9\sin^3\theta_4 + \delta_{10}(\cos\theta_4 - 1)^3 + \delta_{11}\sin^2\theta_4(\cos\theta_4 - 1)^2 + \delta_{12}\sin^3\theta_4(\cos\theta_4 - 1)$$
$$+ \delta_{13}\sin\theta_4(\cos\theta_4 - 1)^3 + \delta_{14}\sin^4\theta_4 + \delta_{15}(\cos\theta_4 - 1)^4 = 0$$

$$\delta_1 = X_1Y_1 - Z_1^2$$
$$\delta_2 = X_1Y_2 + X_2Y_1 - 2Z_1Z_2$$
$$\delta_3 = X_1Y_3 + X_3Y_1 - 2Z_1Z_3$$
$$\delta_4 = X_1Y_4 + X_4Y_1 - 2Z_1Z_4 + X_2Y_3 - X_3Y_2 - 2Z_2Z_3$$
$$\delta_5 = X_1Y_5 + X_5Y_1 - 2Z_1Z_5 + X_3Y_3 - Z_3^2$$
$$\delta_6 = X_1Y_6 + X_6Y_1 - 2Z_1Z_5 + X_2Y_2 - Z_2^2$$
$$\delta_7 = X_2Y_4 + X_4Y_2 + X_6Y_3 + Y_6X_3 - 2Z_3Z_6 - 2Z_2Z_4$$
$$\delta_8 = X_3Y_4 + Y_3X_4 + X_5Y_2 + X_2Y_5 - 2Z_2Z_5 - 2Z_3Z_4$$
$$\delta_9 = X_3Y_5 + X_5Y_3 - 2Z_3Z_5$$
$$\delta_{10} = X_2Y_6 + X_6Y_2 - 2Z_2Z_6$$
$$\delta_{11} = X_5Y_6 + X_6Y_5 - 2Z_5Z_6 + X_4Y_4 - Z_4^2$$
$$\delta_{12} = X_5Y_4 + Y_5X_4 - 2Z_4Z_5$$
$$\delta_{13} = X_4Y_6 + X_6Y_4 - 2Z_4Z_6$$
$$\delta_{14} = X_5Y_5 - Z_5^2$$
$$\delta_{15} = X_6Y_6 - Z_6^2$$

using $\cos\theta_4 = \dfrac{1-x_4^2}{1+x_4^2}$, $(\cos\theta_4 - 1) = \dfrac{-2x_4^2}{1+x_4^2}$ and $\sin\theta_4 = 2x_4/(1+x_4^2)$ the final 8th degree polynomial is obtained as

$$x_4^8(16\delta_{15} + \delta_1 - 2\delta_2 + 4\delta_6 - 8\delta_{10}) + x_4^7(2\delta_3 - 4\delta_4 + 8\delta_7 - 16\delta_{13}) + x_4^6$$
$$(4\delta_1 - 6\delta_2 + 4\delta_5 + 8\delta_6 - 8\delta_{10} + 16\delta_{11} - 8\delta_8) + x_4^5(6\delta_3 - 8\delta_4 + 8\delta_7 + 8\delta_9 - 16\delta_{12})$$
$$+ x_4^4(6\delta_1 - 6\delta_2 + 8\delta_5 + 4\delta_6 - 8\delta_8 + 16\delta_{14}) + x_4^3(2\delta_3 - 4\delta_4 + 8\delta_9) + x_4^2$$
$$(4\delta_1 - 2\delta_2 + 4\delta_5) + x_4(2\delta_3) + \delta_1 = 0$$

Equation (30) gives θ_3. To get S_3 and T we again take suitable dot and cross products:

$$S_3 = \frac{[(R_1^1 - R_1^4) - (\cos\theta_3 - 1)C_{3j}^4] \cdot (D_{3j}^4 \times c_j^1)}{u_{3j}^4 \cdot (D_{3j}^4 \times c_j^1)} \tag{32}$$

$$T = \frac{[(R_1^1 - R_1^4) - (\cos\theta_3 - 1)C_{3j}^4] \cdot (D_{3j}^4 \times u_{3j}^4)}{(c_j^1) \cdot (D_{3j}^4 \times u_{3j}^4)} \tag{33}$$

7b. The R-Sc-C-P, R-Sc-R-C and R-Sc-P-C Cases.

For the R-Sc-C-P, R-Sc-R-C and R-Sc-PC mechanisms, the input-output relationship and the expressions for the intermediate joint variables can be found after simplifying the expression for finite screw displacement and the pair geometry constraints (see reference [24] for details).

For the R-Sc-C-P mechanism the input-output relationship is quadratic. For the R-Sc-R-C mechanism the input-output equation is an eighth degree polynomial in the output tangent half-angle. For the R-Sc-P-C case, the input-output relationship is quadratic. The expression for intermediate joint angles can be found by taking suitable dot and cross products in a manner similar to the R-Sc-C-R case.

8. NUMERICAL EXAMPLE

1. R-Sg-R-R Mechanism.

Given:

$P_1 = 0\hat{i} + 0\hat{j} - 0\hat{k}$; $P_3 = -1\hat{i} + 2\hat{j} + 0\hat{k}$

$P_4 = 2\hat{i} + 3\hat{j} - 0\hat{k}$; $R = 1\hat{i} + 1\hat{j} + 1\hat{k}$

$u_1 = 0\hat{i} + 0.24\hat{j} - 0.97\hat{k}$; $u_3 = 0\hat{i} + 1\hat{j} + 0\hat{k}$

$u_4 = 0\hat{i} + 0.38\hat{j} + 0.95\hat{k}$; $A = 0.707\hat{i} + 0.707\hat{j} + 0\hat{k}$

and θ_1 is the input. Unknowns are θ_3, θ_4, and S_g. The plots of the θ_4 and θ_3 in terms of θ_1 are shown in figures 7 and 8.

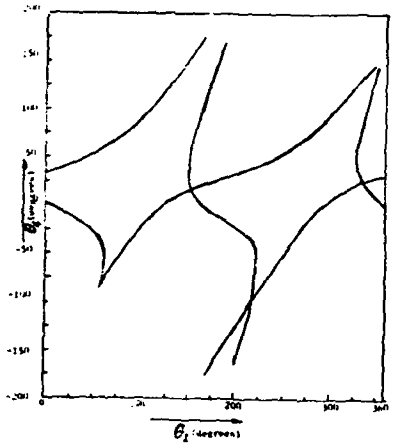


Figure 7. Plot of θ_4 versus θ_1 for R-Sg-R-R Mechanism.

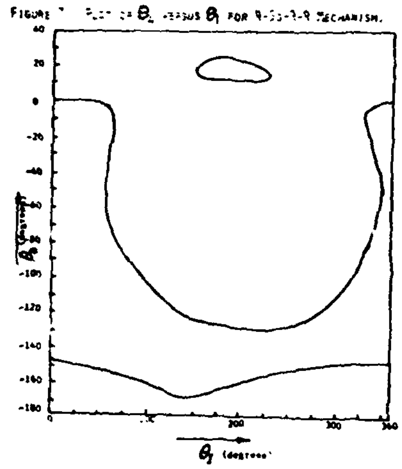


Figure 8. Plot of θ_3 versus θ_1 for R-Sg-R-R Mechanism.

2. R-Sc-C-R Mechanism.

Given:

$P_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$; $P_3 = -1\hat{i} + 2\hat{j} + 0\hat{k}$

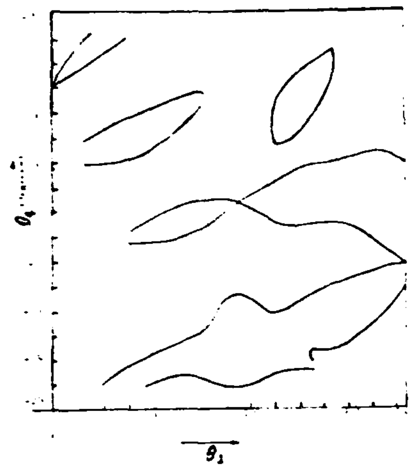
$P_4 = 2\hat{i} + 3\hat{j} + 0\hat{k}$; $R = 1\hat{i} + 1\hat{j} + 1\hat{k}$

$u_1 = 0\hat{i} + 0.707\hat{j} - 0.707\hat{k}$; $u_3 = 0.577\hat{i} + 0.577\hat{j} + 0.577\hat{k}$

$u_4 = -.707\hat{i} + 0.707\hat{j} + 0\hat{k}$; $b = 1\hat{i} + 0\hat{j} + 0\hat{k}$

$c = 0\hat{i} + 0\hat{j} + 1\hat{k}$; $b^* = 1\hat{i} + 0\hat{j} + 0\hat{k}$

and θ_1 is the input. The unknowns are θ_3, θ_4, S_4 and T. The plot of θ_4 in terms of θ_1 is given in Figure 9.



9. CONCLUSION

Displacement equations have been derived for several four-link spatial mechanisms containing sphere-groove and sphere-slotted-cylinder pairs. Velocity and acceleration relationships can be obtained by differentiating the displacement equations. The grooves of these pairs were assumed to have straight axial centerlines. However, a more generalized groove may be one where the centerline is a spatial curve. The authors are working on the analysis of these and also of other three, four, five and six link mechanisms containing other higher pairs. The expected result of the work will be reported in forthcoming papers.

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ANALYSIS OF SPATIAL MECHANISMS CONTAINING HIGHER PAIRS

BY

ASHITAVA GHOSAL

A THESIS PRESENTED TO THE GRADUATE COUNCIL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE

UNIVERSITY OF FLORIDA

1982

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TABLE OF CONTENTS

CHAPTER		PAGE

APPENDICES PAGE

Abstract of Thesis Presented to the Graduate Council
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Master of Science

ANALYSIS OF SPATIAL MECHANISMS
CONTAINING HIGHER PAIRS

By

Ashitava Ghosal

August 1982

Chairman: George N. Sandor
Major Department: Mechanical Engineering

In this work, results of the investigation dealing with analysis of spatial mechanisms containing higher pairs are presented. Complete analytical expressions for the position, velocity and acceleration analysis for several three-, four-, and five-link spatial mechanisms containing higher pairs are presented. Also presented are computer programs for the analysis of some of these mechanisms.

A higher pair as distinct from a lower pair allows more degrees-of-freedom between its elements. The kinematic analysis of spatial mechanisms containing higher pairs is based on the concept of finite screws and pair geometry constraints. Expressions for finite screws and their derivatives have been developed and expressed in a shorthand notation. The pair geometry constraints for sphere-plane, sphere-groove, sphere-slotted-cylinder

and cylinder-plane higher pairs are presented. Using these pair geometry constraints, and the finite screws and its derivatives, several mechanisms containing the above mentioned higher pairs have been analyzed for position, velocity and acceleration.

Computer programs are given for the analysis of some of the typical three-, four-, and five-link mechanisms. The use of the programs are demonstrated in the examples in Chapter VII.

George N. Sandor
Chairman

OPTIMIZATION OF SPATIAL MECHANISMS

By

CHARLES FREDERICK REINHOLTZ

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1983

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TABLE OF CONTENTS

Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

OPTIMIZATION OF SPATIAL MECHANISMS

By

Charles Frederick Reinholtz

August, 1983

Chairman: George N. Sandor
Cochairman: Joseph Duffy
Major Department: Mechanical Engineering

The material in this dissertation can be effectively divided into two subtopics: philosophy of optimal mechanism design, and optimization of dyad-based spatial mechanisms.

The first subtopic, philosophy of optimal mechanism design, is intended to be general in nature, applying to all types of mechanisms, both higher and lower pair, and both planar and spatial. This is covered in Chapters One through Three. Chapter One examines past approaches to mechanism optimization. Chapter Two is a brief review of optimization theory, particularly as it applies to mechanism optimization. Chapter Three draws upon the insights gained in the first two chapters to formulate a general approach to the mechanism optimization problem.

The second subtopic of this dissertation, optimization of dyad-based spatial mechanisms, is covered in Chapters

Four through Seven. This is actually a rather limited example of applying the philosophy developed in the first three chapters. Nevertheless, the mechanisms treated in this section are believed to represent some of the most useful motion generating spatial mechanisms, and, therefore, those for which improved design theories are most urgently needed. In Chapter Four, closed-form synthesis equations are derived for dyads containing revolute (R), spheric (S) and cylindric (C) pairs. Chapters Five and Six present detailed examples of the optimization of the four-link RCCC and five-link RSSR-SC and RSSR-SS mechanisms. Finally, Chapter Seven outlines procedures for the optimization of other dyad-based spatial mechanisms, and offers suggestions for further research.

KINEMATIC SYNTHESIS AND ANALYSIS
OF
THREE-LINK SPATIAL FUNCTION GENERATORS WITH HIGHER PAIRS

BY

MANUEL VENADAS HERNANDEZ JR.

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
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TABLE OF CONTENTS

APPENDICES | Page

Abstract of Dissertation Presented to the Graduate Council
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

KINEMATIC SYNTHESIS AND ANALYSIS OF THREE-LINK SPATIAL FUNCTION GENERATORS WITH HIGHER PAIRS

By

Manuel Venadas Hernandez Jr.

April 1983

Chairman: Dr. George N. Sandor
Major Department: Mechanical Engineering

Function generation synthesis of spatial mechanisms with only three links is achieved by employing higher pairs (sphere-plane (Sp), cylinder-plane (Cp) and sphere-groove (Sg) pairs) to constrain the motion of two links.

This dissertation shows the methods and procedures for obtaining the equations for multiply-separated-precision point (MSP) synthesis for four spatial function generators - R-Sp-R, R-Sp-P, R-Cp-C and R-Sg-C.

Higher pair constraint equations in vector form are utilized to obtain closed-form solutions for the different synthesis cases of various numbers of positions and specified and unknown parameters. The method of elimination is used extensively to solve the resulting non-linear systems of equations.

Kinematic synthesis and analysis of the four spatial function generators is performed in vector notations and with screw displacements in vector form. Explicit equations are also obtained from the investigation of the transmission characteristics of these mechanisms.

The synthesis procedures just completed were then augmented by developing design criteria for the pair elements to assure range of mobility and avoidance of interference.

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